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Modelling the Term Structure of Interest Rates and Volatility in China

Li, Songzhuo

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Modelling the Term Structure of Interest Rates and Volatility in China

Songzhuo Li

A thesis submitted for the degree of Doctor of Philosophy
University of Bath
Department of Economics

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Table of Contents

Abbreviation	VIII
Abstract	IX
Chapter I. Introduction.....	1
Chapter II. A Literature Review on Term Structure of Interest Rates Modelling	5
II.1. Introduction.....	5
II.2. Literature Worldwide	5
II.2.1. Yield Curve Smoothing Models.....	5
II.2.2. No-arbitrage Affine Factor Models.....	10
II.2.3. Macro-Finance Models	24
II.2.4. Markov Regime-Switching Models.....	32
II.3. Background of Chinese Government Bond Market.....	34
II.3.1. Review of History	35
II.3.2. Overview of Chinese Government Bond Market	36
II.3.3. Monetary Policy in China	40
II.3.4. Achievements of Chinese Interest Rate Liberalization.....	42
II.3.5. Problems in Chinese Government Bond Market	44
II.4. Literature in China	47
II.4.1. Yield Curve Fitting	47
II.4.2. Dynamic Modelling	48
Chapter III. Chinese Government Yield Curve Analysis with Cyclical Mean	50
III.1. Introduction and Motivation	50
III.2. Fourier Models and Methodologies	52
III.2.1. The One Term Fourier Model	52
III.2.2. The Two Terms Fourier Model	55
III.2.3. Estimation Method.....	56
III.2.4. Prediction Approach.....	56

III.3. Empirical Analysis	57
III.3.1. Data Description	57
III.3.2. In-sample Fitting of Yield Curve	59
III.3.3. Out-of-Sample Forecasting	65
III.3.4. Comparison with the Study in U.S.	74
III.4. Summary and Conclusion	76
Chapter IV. Interactions between Chinese Government Bond Yields and the Economy	
	79
IV.1. Introduction.....	79
IV.2. Model and Estimation Method.....	82
IV.2.1. Model Construction	83
IV.2.2. Estimation Methods	87
IV.3. Data Description	90
IV.4. Empirical Results	94
IV.4.1. Parameter Estimation	94
IV.4.2. Extraction of Latent Factors.....	97
IV.4.3. Impulse Response Function	101
IV.4.4. Variance Decomposition	103
IV.5. Summary and Conclusion	107
Chapter V. Regime-Switching Diffusion Models of Short-Term Interest Rate	
	109
V.1. Introduction.....	109
V.2. Data and Model.....	111
V.2.1. Data Description	111
V.2.2. Models of Short Term Interest Rate	114
V.2.3. Maximum Likelihood Estimation of Regime-switching Models	117
V.3. Empirical Analysis	119
V.3.1. Parameter Estimations	119
V.3.2. Model Comparison.....	124

V.4. Conclusions.....	127
Chapter VI. Summary and Conclusions	129
Reference	131

List of Tables

Table II-1 Yield curve smoothing methods used in various central banks	8
Table II-2 Yield Curve Smoothing Models Mentioned in this Chapter	9
Table II-3 No-Arbitrage Affine Factor Models Mentioned in this Chapter	23
Table II-4 Macro-Finance Models Mentioned in this Chapter	31
Table II-5: Achievements of Interest Rates Liberalization in China.....	43
Table III-1: Summary statistics of Chinese zero-coupon yields	58
Table III-2: Parameter mean from in-sample Fitting	60
Table III-3: Goodness of Fit of Yields with Each Maturity	61
Table III-4: One-day Ahead Forecasting Errors in Period 1	67
Table III-5: One-week Ahead Forecasting Errors in Period 1	67
Table III-6: One-month Ahead Forecasting Errors in Period 1.....	68
Table III-7: One-day ahead forecasting errors in period 2.....	70
Table III-8: One-week ahead forecasting errors in period 2	70
Table III-9: One-month ahead forecasting errors in period 2	71
Table III-10: One-day ahead forecasting errors in period 3	73
Table III-11: One-week ahead forecasting errors in period 3	73
Table III-12: One-month ahead forecasting errors in period 3	74
Table IV-1: Descriptive Statistics of Chinses Treasury Yields	93
Table IV-2: Principle Component Analysis of Chinese Treasury Yields	93

Table IV-3: Estimates of Matrix A and Factor Means.....	95
Table IV-4: Estimates of Covariance Matrix	96
Table IV-5: Summary Statistics for Measurement Errors of Yields.....	97
Table IV-6: Descriptive Statistics of the Extraction of Latent Factors	98
Table IV-7: Variance Decomposition of Latent Factors with Monthly Data	104
Table IV-8: Variance Decomposition of Macroeconomic Variables for Monthly Data	106
Table V-1 Summary statistics of 3-month treasury yields	111
Table V-2 Single-regime and regime-switching Models employed in this study	117
Table V-3 Parameter estimations in single-regime models.....	119
Table V-4 Parameter estimations in regime-switching models.....	121
Table V-5. Ljung-Box statistic for serial correlation of the squared residuals out to i lag.....	123
Table V-6. Likelihood Ratio Tests.....	125

List of Figures

Figure II-1: Issuance volume of Chinese government bond market.....	37
Figure II-3: Investor Profile in December 2015	40
Figure II-4: Evolution of Reserve Requirement Ratio in China from 1987 to 2016 ...	41
Figure II-5: Chinese and Japanese Government Bond Market Size in Percentage of GDP	45
Figure II-6: Maturity Profile of Chinese Government Bond Issued in 2015	46
Figure II-7: Government Bond Turnover Ratio of China and Japan	46
Figure III-1: Estimated Parameters from Vasicek Model within the Whole Sample...	62
Figure III-2: Estimated Parameters from Fourier within the Whole Sample.....	63
Figure III-3: Estimated Parameters from Vasicek within Post-crisis Period	64
Figure III-4: Estimated Parameters from Fourier within Post-crisis Period	65
Figure III-5: 3-D Plot of Yield Curve of the First Forecasting Period (12/11/2008-12/25/2009).....	66
Figure III-6: 3-D Plot of Yield Curve of the Second Forecasting Period (12/11/2008-12/25/2009).....	69
Figure III-7: 3-D Plot of Yield Curve of Forecasting Period 3 (06/16/2014-06/30/2015).....	72
Figure III-8: Estimated Parameters from Vasicek in U.S.....	75
Figure III-9: Estimated Parameters from Fourier in U.S	76
Figure IV-1: Plots of Parameter Loadings	84
Figure IV-2: Chinese Government Yield Curve (03/01/2006-04/24/2015)	92

Figure IV-3: Chinese Spot Yields (03/01/2006-04/24/2015)	92
Figure IV-4: Extractors of Latent Factors from Macro-Finance Model	99
Figure IV-5: Extracted level factors from yields-only and macro-finance models and data-based counterpart	100
Figure IV-6: Extracted slope factors from yields-only and macro-finance models and data-based counterpart	100
Figure IV-7: Extracted curvature factors from yields-only and macro-finance models and data-based counterpart	101
Figure IV-8: Impulse Response Functions of Yield Factors and the Macro Variables	103
Figure V-4: Smoothed Probabilities in Regime 1 in Regime-switching Models.....	126

Abbreviation

CDB	China Development Bank
CHIBOR	China Interbank Offered Rate
CPI	Consumer Price Inflation
CSDS	China Central Depository and Clearing Corporation
EXIMCH	Export-Import Bank of China
GARCH	Generalized Autoregressive Conditional Variance
GDP	Gross Domestic Product
OTC	Over-the-Counter
PBC	People's Bank of China
RMB	Renminbi
RR	Required Ratio
SLO	Short-Liquidity Operations
SHIBOR	Shanghai Interbank Offered Rate
VAR	Vector Autoregression

Abstract

The purpose of this study is to examine the dynamic behaviour of the Chinese yield curve and the short rate volatility. It consists of an element of literature reviews on the term structure of interest rates and three empirical chapters. The empirical studies discuss the dynamics of Chinese yield curve, the interactions between Chinese yield curve and economy and the volatility in the Chinese short-term interest rate.

First, we employ the Fourier model to estimate the term structure of Chinese interest rates, following Moreno, Novales and Platania (2013). The Fourier model is an extension of Vasicek model by imposing a Fourier series to describe the long-run mean. The Fourier model provides better approximation and prediction of the dynamics of Chinese yield curve than the Vasicek model, especially on the short end. We conclude that the Fourier assumption does help to capture the volatility of Chinese yield curves, as the Chinese yield curve is found to behave cyclically.

Second, we construct and estimate the Nelson-Siegel form macro-finance model based on Chinese market, following Diebold, Rudebusch and Aruoba (2006). Bidirectional causality is found, however the yield curve effect on the macroeconomy is relatively weak compared to the reverse influence. In the long-term horizon, both the inflation rate and real activity as approximate by industrial production, can explain more than 30 percent of the variation of yield curve.

Finally, we examine the dynamic behaviour of Chinese short rate in frame of Conley, Hansen, Luttmer and Scheinkman (1997). Four one-factor diffusion models and four Markov regime-switching extensions are compared. We find that incorporating regime-switching can largely improve in-sample fitting to data and also help to capture the volatility clustering and fatter tail. The nonlinearity of drift term seems of importance on capturing the movement of Chinese short rate.

Chapter I.Introduction

Term structure of interest rate describes the relationship of interest rates on zero-coupon bond with different terms to maturity. It plays a central and fundamentally essential role in financial economics, both theoretically and empirically. The central bank conducts monetary policy by controlling the interest rate at the short end of yield curve directly in order to achieve their goals of stabilization, while longer term yields serves as an indicator of the market participants' expectations and responses to future economic conditions. In addition, the information contained in the term structure of interest rates is valuable to both policy makers and investors. A precise extraction of information from the yield curve provides one of the most important factors for monetary policy establishment, pricing and hedging of interest rate derivatives and risk management of interest rate contingent portfolios.

While the importance of term structure of interest rate has resulted in tremendous studies on the modelling of interest rates with empirical evidence from the U.S. and other advanced economies, studies based on emerging market is rare. In particular, attentions on the modelling of Chinese term structure of interest rates is limited and lag behind developed countries, because of the regulated interest rates and immature market within a long history in China.

The modelling of interest rate in China is worthy of study for three reasons. First, the Chinses government bond market grows dramatically in recent years and is large in size with 35.04 trillion RMB outstanding until December 31, 2015. It has become the third largest government bond markets globally after the U.S. and Japan. Also, as the second largest economy in the world, the impact of China to the global financial markets is only likely to increase. Last but definitely the most important, China has been transferring to a more market oriented financial system with significant financial innovations, and the progress which has been made in financial reforms promotes more liberalized interest rates. This forms the basis for the empirical study of interest rate modelling in China.

Most existing studies on Chinese term structure of interest rates are confined to the static curve fitting at a given date, lacking analysis on the overall dynamic movements of the yield curve through a period. This thesis focuses on examining the dynamic behaviour of Chinese interest rates and the volatility. We are interested in that if the models which performs well in the U.S. are suitable to China and what are the similarities and differences between the movements of Chinese yield curve or short rate and those in the U.S. We attempt to capture important features of Chinese interest rates dynamics to provide valuable information to both policy makers and market participants.

With a comprehensive review of literature, studies on Chinese term structure of interest rates are summarized for comparison based on a brief overview of Chinese government bond market. The empirical analysis investigates the Chinese interest rate dynamics from three aspects, which are dynamic movements of yield curve, relationship between yield curve and macroeconomy and the short-term rate volatility.

In chapter 2, a general literature review is given on the term structure of interest rate modelling. Widely used yield curve smoothing techniques, no-arbitrage affine factor framework and the popular interdisciplinary macro-finance models are introduced. In addition, the background of Chinese government bond market is summarized. After a review of the development history, an overview on Chinese government bond market is described from various aspects. Both monetary policy and achievements in the interest rates liberalisation are discussed. We also point out some major problems of Chinese government bond market. Under the specific background, existing studies on Chinese term structure of interest rates are reviewed.

In Chapter 3, we introduce a Vasicek extension to the Chinese treasury yields modelling which incorporates a Fourier series describing the long-run mean. Both in-sample fitting and out-of-sample forecasting performance are investigated and compared with similar study based on the U.S. market. This study tries to capture the cyclical behavior of interest rates by assuming cyclical long-run equilibrium mean while previous studies set it as a constant. This assumption is proved to be useful in the empirical results. In addition, we derive the theoretical two-term Fourier extension model which should give

more flexibility to the model in describing the dynamics of term structure of interest rates. But for computation issues, we leave the empirical analysis to future work.

Chapter 4 explores the interactions between yield curve and the economy in China. The yield curve is connected with inflation and real activity via a macro-finance model, which is a macro extension of dynamic Nelson-Siegel framework. The estimation method employed is the one-step maximum likelihood approach using Kalman filter, rather than the conventional two-step ordinary least squares regression method. The state-space system enables us to investigate the interactions between the yield curve and macroeconomic variables by using the techniques of impulse response function and variance decomposition, and extract latent yield factors to capture the dynamic behavior of yield curve directly. The main differentiation of this research from other related studies is that macroeconomic variables are incorporated to term structure model directly which gives us an easier way to explore the bidirectional influences between them. Similar studies based on emerging market especially China, is rare.

The risk-free short-term interest rate plays a fundamental role in the financial economics. It is an instrument of monetary policy and consider to be reference rate for asset pricing in terms of excess returns. In addition, the default-free short rate composes the short end of the yield curve, therefore the pricing of fixed-income securities and derivatives at all the maturities are associated with it. After the discussion of the dynamic variation on the whole yield curve, we pay a special attention on the short end of Chinese yield curve.

Chapter 5 examines the dynamic behaviour of Chinese short-term interest rate. The inter-bank 3-month treasury yields are used as proxy to measure the market short rate. The existing studies on Chinese short rate indicates important features of it, including mean-reversion and volatility clustering. Based on these findings, we incorporating Markov regime-switching to the CHLS (1997) framework which nests widely used diffusion models in one system. This framework enables us to compare the popular Vasicek, CIR, CKLS and CHLS models directly and test the level effect, nonlinearity on drift term and regime-switching specification at once. The models are estimated by

maximum likelihood method following Hamilton (1988) 's algorithms. This study use a variety of models to evaluate the movements of Chinese short-term interest rate under one common framework and is beneficial from the nesting-system of models and the up-to-date data.

In chapter 6, the results and findings of this thesis is summarized and suggestions for future work are outlined. The main findings of this thesis could be summed up as bellows. In the first essay, significant impact of the 2007 financial crisis is found on the Chinese interest rates. In addition, the Fourier model proves to have better performance in both approximation and prediction than the benchmark model. Bidirectional causality is found in the second essay, where the macroeconomy effect on the yield curve is stronger than the reverse influence. The macro variables are found to explain more than 30 percent of the variation of yield curve in the long-term horizon. In the last essay, we find that incorporating regime-switching can largely improve in-sample fitting. The least restricted regime-switching model shows best in-sample fitting performance among the others.

In all, this research contributes to the literature along four dimensions as follows. First, existing studies mostly focus on the static yield curve fitting at a given time in China, we investigate the dynamic movements through the whole yield curve during a period. Second, this is one of the earliest research providing not only in-sample fitting to data but also out-of-sample forecasting on Chinese term structure modelling. Third, this is a pioneer work which investigates the bidirectional influence between yield curve and macroeconomic variables in China. Furthermore, innovative methods are developed in this thesis, including Fourier extension on capturing cyclical behaviour, Kalman filter in one-step estimation and CHLS framework for nesting various models in one system. At last, the popular models perform well in mature markets are employed to Chinese market, so that we can study the evolvement of the Chinese government bond market by summarizing the similarities and differences on interest rates behaviour in the U.S. and China.

Chapter II. A Literature Review on Term Structure of Interest Rates Modelling

II.1. Introduction

The term structure of interest rates, also known as yield curve, describes the relationship between yields to maturity on zero-coupon Treasury bonds and their corresponding time to maturities at a given time. Due to its fundamental and important role in both finance and economics, the term structure of interest rate has been extensively examined. There is huge amount of literatures which tries to investigate the movement of yield curve theoretically and empirically. In section 2, a worldwide literature review is given on the term structure of interest rate modelling with focus on three directions. The yield curve smoothing or yield curve fitting is a technique to extract the interest rates from bond prices by using statistical methods. It solves the primary problem in the yield curve analysis. As to the dynamic rather than static yield curve model, we review a strand of no-arbitrage affine factor models. The affine framework with arbitrage-free restriction is a generalized form which can nest many classic term structure models. In addition, a brief review of macro-finance modelling is presented. The macro-finance is a new but very popular area in recent years and it builds a bridge between finance and economics by combining the yield curve with macroeconomy. Section 3 presents the background on Chinese government bond market. With a summary of Chinese government bond market, another literature on the study of term structure of interest rate on China is reviewed.

II.2. Literature Worldwide

II.2.1. Yield Curve Smoothing Models

The primary problem in yield curve analysis lies in that the interest rates which form the yield curve are rarely observable directly. In financial market, the pure discount government bonds prices which can be used to compute the yield to maturity are mostly available up to one-year time to maturity. However, the fact that the coupon-bearing

bond is a portfolio of pure discount bonds, make it possible to extract the zero-coupon rates from the observable prices of coupon-bearing government bonds. Hence, numerous statistical techniques are developed in an attempt to extract zero-coupon rates from coupon bonds data. These models, so-called yield curve smoothing models, are based-on curve-fitting techniques which fit a continuous function to a set of discretely observed data points without any theoretical foundation. The yield curve smoothing models could be roughly summarized to two categories which are splines and parsimonious types according to the approximation function.

Spline-based Models

The spline-based models employ spline function which is piecewise-defined by polynomial functions to fit the yield curve. The basic idea is that any continuous functions within a closed interval, can be approximately expressed by selecting an arbitrary polynomial function. All the individual segments of these functions can be combined at knot points to form a continuous and smooth yield curve.

This type of model was firstly proposed by McCulloch (1971, 1975) who employs polynomial spline functions to approximate the discount function. In his models, the price of a bond with par value 100 is given in continuous-coupon form as

$$p = 100\delta(m) + c \int_0^m \delta(\mu) d\mu \quad (2.1)$$

where c is coupon rate, m is terminal maturity is date and $\delta(\mu)$ is the discount function. In order to estimate the discount function from observations on the prices of n bonds by linear regression, the discount function is expressed as the sum of a constant and a linear combination of postulated functions. A quadratic spline function is used to estimate the discount function in McCulloch (1971). The order of the estimation function is increased to a cubic spline in McCulloch (1975) so as to avoid the “knuckles” effect found in the previous model. But the drawback of this method is that the forward rate can be negative since the discount function is not restricted as non-increasing.

In order to overcome the issue of negative forward rate in McCulloch (1975), Schaefer

(1981) introduces the Bernstein polynomials to fit the curve of discount function and the constraints of non-negative and monotonic non-increasing discount function is incorporated. The Bernstein polynomial functions provide better approximations to the derivatives which is essential in smoothing model, since the first order derivative of the discount function is the forward rate.

Vasicek and Fong (1982) suggest an exponential spline to estimate the discount function instead of polynomial splines. They argue that the exponential splines provide better local fit and slope approximation to the discount function than the polynomial spline methods. Complex nonlinear estimation avoidance and asymptotic property of the yield curve are the superiority of this model.

Fisher, Nychka and Zervos (1995) propose a smoothing splines method rather than the regression splines in previous studies, with a roughness penalty to extract the forward rate curve. The number of parameters to be estimated in this model is not pre-specified as in the regression spline models. Instead, a generalized cross validation technique is employed to determine the effective number of parameters and location of the knot points optimally.

Although the McCulloch (1975) model provide both good in-sample and out-of-sample estimation to the bond prices, the forward rate curve extracted from this model is found having an oscillate behaviour. In Fisher, Nychka and Zervos (1995), the oscillatory is decreased by using the roughness penalty, but the goodness-of-fit is reduced as well. At the meantime, this model is proven to misprice at the short end by Bliss (1997). In order to overcome these problems, Waggoner (1997) introduces a variable roughness penalty to Fisher, Nychka and Zervos (1995)'s method instead of a constant. The roughness penalty consists of a small one in the short end of the forward rate curve and a larger one on the long-term maturities to ensure flexibility within different maturities and more accurate fit.

Parsimonious Models

Rather than relying on piecewise polynomial spline functions, the parsimonious

smoothing model specifies a single-piece function over the entire maturity to fit the yield curve. In this type models, parameters are estimated by minimizing the squared deviations of the theoretical prices from observable data and models differ on the selection of the single-piece function.

The most popular model belonging to this category is the Nelson and Siegel (1987) which fits the forward rate curve at a given date with a three-component exponential approximation. The forward curve is assumed to be the solution to a second order differential equation with equal roots for spot rates in the approximation. The parametric curves in this model are flexible enough to describe a whole family of observed term structure shapes. The zero-coupon yields with different maturities could be given as a function in terms of three unobserved factors β_1 , β_2 and β_3 at any point of time:

$$y(\tau) = \beta_1 + \beta_2 \left[\frac{1 - \exp(-\lambda\tau)}{\lambda\tau} \right] + \beta_3 \left[\frac{1 - \exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau) \right] \quad (2.2)$$

where $y(\tau)$ is the zero-coupon yields, τ is maturity and parameter λ is the exponential decay rate.

Svensson (1994) proposed a four-factor model by adding a second hump term to Nelson and Siegel (1987). The spot rate with maturity τ can be given as:

$$y(\tau) = \beta_1 + \beta_2 \left[\frac{1 - \exp(-\lambda_1\tau)}{\lambda_1\tau} \right] + \beta_3 \left[\frac{1 - \exp(-\lambda_1\tau)}{\lambda_1\tau} - \exp(-\lambda_1\tau) \right] \\ + \beta_4 \left[\frac{1 - \exp(-\lambda_2\tau)}{\lambda_2\tau} - \exp(-\lambda_2\tau) \right] \quad (2.3)$$

Although both models are lack of theoretical support, they are widely used by central banks and market participants to model the term structure of interest rates. As shown in Table II-1, nine out of thirteen central banks reported to use either the Nelson and Siegel (1987) or the Svensson (1994) extension to generate zero-coupon yield curves according to Bank of International Settlements (BIS 2005).

Table II-1 Yield curve smoothing methods used in various central banks

Central Bank	Estimation Method
Belgium	Svensson or Nelson-Siegel
Canada	Merrill Lynch Exponential Spline
Finland	Nelson-Siegel
France	Svensson or Nelson-Siegel
Germany	Svensson
Italy	Nelson-Siegel
Japan	Smoothing splines
Norway	Svensson
Spain	Svensson, Nelson-Siegel (before 1995)
Sweden	Smoothing splines and Svensson
Switzerland	Svensson
United Kingdom	VPR ¹ , Svensson (Jan. 1982 to Apr. 1998)
United States	Smoothing splines

Source: BIS (2005)

Table II-2 Yield Curve Smoothing Models Mentioned in this Chapter

Category	Author	Year	Specification
Spline-based	McCulloch	1971	Employs polynomial spline functions to approximate the discount function
	McCulloch	1975	A cubic spline is used to avoid the “knuckles” effect

¹ VPR: variable penalty roughness. It is a method implemented by the Bank of England allowing the roughness parameter to vary with maturity.

	Schaefer	1981	Uses Bernstein polynomials and restricts the discount function to be non-negative and monotonic non-increasing to secure positive forward rates
	Vasicek and Fong	1982	Exponential spline is found to provide better local fit and slope approximation
	Fisher, Nychka and Zervos	1995	Uses smoothing splines method rather than the regression splines in previous studies, with a roughness penalty
	Waggoner	1997	Introduces variable roughness penalty instead of constant roughness penalty
Parsimonious	Nelson and Siegel	1987	Fits the forward rate curve with a three-component exponential approximation at a given date, so the yields with different maturities could be given as a function in terms of three unobserved factors
	Svensson	1994	Introduces a four-factor model by adding a second hump term to Nelson and Siegel

II.2.2. No-arbitrage Affine Factor Models

As given the frequently-used yield curve smoothing models, we introduce the dynamic no-arbitrage affine factor models of the term structure of interest rate in this section. In these models, affine condition and cross-equation restrictions are incorporated to ensure no arbitrage opportunity exists. Bonds with different maturities are traded at the same time. Risk-averse investors demand compensation to bear the risk of bonds with long maturities since they have higher risk than short-term bond. Arbitrage means it is

possible to have a portfolio that is risk free and definitely offers positive payoffs. Arbitrage opportunity in bond market exists unless long-term yields are risk-adjusted expectations of average future short rates. Therefore, the cross section yields movements are tied together which presents as cross-equation restrictions in a yield-VAR. The necessity of encountering cross-equation restrictions implied by no arbitrage are explained as follows. Firstly, the yields dynamics are consistent with these restrictions, since in real world the arbitrage opportunity are traded away immediately due to high liquidity of the markets. Secondly, the restrictions are needed to model the time-varying term premium which is essential in term structure. Thirdly, in emerging markets, bonds are traded just with limited available maturities and the missing yields could be recovered from a set of other yields under the consistence of the arbitrage-free restrictions. Last but not least, the restrictions improve the efficiency of estimation compare to the large number of parameter estimation in unrestricted regressions. Also, the no-arbitrage affine factor models allow us to model the term premium explicitly.

In this section, we begin with introducing the pricing kernel and affine condition and two simple single-factor models, the Vasicek (1977) and Cox-Ingersoll-Ross (1985) are given. Then we generalize the discussion to multifactor cases. A general form affine class model from Duffie and Kan (1996) is presented with special attention on time-varying premia. Then the canonical affine and essentially affine models are discussed as extension of the no-arbitrage multi-factor affine models.

The majority literature presents term structure models by using continuous-time framework where stochastic calculus reigns and partial differential equations spit fire. The literature review on macro-finance models in next section calls for the discrete-time framework and in order to keep consistency, we present all the no-arbitrage dynamic factor models in discrete-time form. The reason why the discrete-time models are much more suitable for term structure models with macroeconomic variables is that the market data can be sampled at different frequencies, while the macroeconomic variables are typically monthly data. Moreover, the estimation of discrete-time models is more affordable compared with continuous-time models. For instance, Backus et al. (1998) shows the estimation of some simple one-factor and multi-factor affine models

within a discrete-time framework. Ang and Piazzesi (2003) use a discrete-time VAR to model the dynamic of term structure with macro variables. The VAR (1) model used by Rudenbusch and Wu (2004) can easily be expressed as a Gaussian affine term structure model. As a result, a discrete-time setting may be much more convenient for macro-finance term structure models.

Pricing Kernel and Affine Condition

As the asset pricing theory claims, under assumption of non-arbitrage opportunity, a positive random variable M which is referred to as pricing kernel or stochastic discount factor must exist. Consider the optimization problem of an investor who would like to maximise the expectation of a utility function:

$$\max_{c_t} E_t[\sum_{t=0}^{\infty} \delta^t U(c_t)] \quad (2.4)$$

subject to the constraint

$$c_t = f(k_t) - (\beta + n)k_t - (k_{t+1} - k_t) \quad (2.5)$$

where c_t is the consumption during time period t , $U(c_t)$ is the utility of consumption during time period t , δ is the time discount factor, $f(k_t)$ is the output per unit of labour, k_t is the capital intensity at time t , n is the population growth and $\beta \in [0,1)$ is the depreciation rate of the capital. The expectation is taken conditionally on the history of the information available at time t .

Applying the first order condition, we get the fundamental relation for bond pricing:

$$1 = E_t[(1 + R_{t+1})M_{t+1}] \quad (2.6)$$

where the pricing kernel

$$M_{t+1} = \delta \left(\frac{U'(c_{t+1})}{U'(c_t)} \right) \quad (2.7)$$

We will use the pricing kernel method to price zero-coupon bond in term structure

models. P_t^n denotes the price of a zero-coupon bond at time t with n period left to the maturity. y_t^n is the spot rate at time t on the n -period zero-coupon bond. By assuming the zero-coupon bond pays off one unit at maturity, the relationship between the bond price and the yield to maturity becomes:

$$y_t^n = -\frac{1}{n} \log P_t^n \quad (2.8)$$

Thus, the short rate could be expressed as:

$$r_t = y_t^1 = -\log P_t^1 \quad (2.9)$$

The bond price for the next period $t + 1$ is denoted as P_{t+1}^{n-1} with $n - 1$ period left to maturity. The one-period return on the n -period zero-coupon bond purchased at time t and sold at time $t + 1$ is defined as R_{t+1}^n . So the holding-period return is

$$(1 + R_{t+1}^n) = \frac{P_{t+1}^{n-1}}{P_t^n} \quad (2.10)$$

Then substitute the equation (2.10) into equation (2.6) and we get this result on the price of the n -period zero-coupon bond:

$$P_t^n = E_t[P_{t+1}^{n-1} M_{t+1}] \quad (2.11)$$

Assuming the zero-coupon bond pays off one unit at maturity with the bond price $P_t^0 = 1$, we could calculate the bond prices recursively and get the equation of the price of the n -period zero-coupon bond as the expected product of the pricing kernels:

$$P_t^n = E_t[M_{t+1} M_{t+2} \dots M_{t+n}] \quad (2.12)$$

In order to keep the arbitrage opportunity out, we have to ensure the pricing kernel to be positive. Thus, we take natural logarithm of the pricing kernel and model $\log M_{t+1}$ instead:

$$m_{t+1} = \log M_{t+1} \quad (2.13)$$

Assuming the pricing kernel M_{t+1} is conditionally lognormal distributed and the bond prices P_t^n are jointly lognormal with M_{t+1} , the pricing equation is obtained by taking the logarithm of equation (2.11):

$$\log P_t^n = E_t[m_{t+1} + \log P_{t+1}^{n-1}] + \frac{1}{2} \text{var}_t[m_{t+1} + \log P_{t+1}^{n-1}] \quad (2.14)$$

For a traditional no-arbitrage affine factor model, three components are included to model the term structure of interest rates. The first component is the transition equation which gives a description of how the latent state variables x_t relates to the bond prices dynamically. The second equation relates the one-period short rate to the latent factors. Finally, the pricing kernel equation describes the relationship between the term premium, the shocks and the latent factors.

In such a no-arbitrage affine factor model system, the logarithm of bond price, and hence the nominal bond yield is an affine function of some latent factors where affine means a linear function plus a constant. The affine models are special class of the term structure models which describe the yield as:

$$y_t = \rho_0 + \rho' x_t \quad (2.15)$$

where y_t is the nominal bond yield, x_t is an n -dimensional vector of the state variables, ρ is a constant n -dimensional vector and ρ_0 is a constant.

Tractability is the main advantage of affine models. Affine term structure models give tractable solutions for bond yields which is much simpler and convenient than using the Monte Carlo methods or solution methods for PDEs to compute the yields. We will introduce the early steps of the affine models where the riskless rate is the only state variable in the models namely single-factor models and then the multi-factor models with extensions on the number of state variables which are more complete affine models on bond yields. Both two classes of models give closed-form solutions for bond yields.

Single-factor Models

In single-factor models, the short rate is assumed to be the only factor which means the term structure of interest rates is driven by the single state variable typically associated with the short rate. We will present two popular short rate models constructed by Vasicek (1977) and Cox, Ingersoll, and Ross (1985) (hereafter CIR) setting in discrete-time framework although they are originally given in the continuous-time form.

Vasicek Model

The Vasicek (1977) model is one of the earliest no-arbitrage models and based upon the idea of mean reverting feature of interest rates. The short rate is associated with the single state variable x_t whose dynamics follows a first-order autoregression process AR (1):

$$x_{t+1} - x_t = k(\theta - x_t) + \sigma \varepsilon_{t+1} \quad (2.16)$$

where k, θ and σ are non-negative constants, $\varepsilon_{t+1} \sim N(0,1)$ and k controls the mean reversion of the state variable process. Since equation (2.16) could be rewritten as $x_{t+1} = k\theta + (1 - k)x_t + \varepsilon_{t+1}$, we assume that $0 < k < 1$ and then the state variable is expected to revert to the “long run equilibrium level” θ . If $k = 0$, the state process is a random walk and that leads to a possibility for interest rates to be arbitrarily large even to infinity which is not reasonable in the economic world. We will prove that the single factor is just the short rate in this model.

So under the assumption of mean reversion, if interest rate is larger than the long run equilibrium level ($x_t > \theta$), then the interest rate will be forced to decline to approach the level θ as the drift is negative. In contrast, interest rate will be driven to grow up to approach the equilibrium level when it is less than level θ ($x_t < \theta$), as the drift keeps being positive. The economic phenomenon, that as the movements of interest rate changing over time, it is always hauled back to some average level of interest rates could be reflected by the mean reversion assumption of interest rate in the Vasicek model.

An economic argument which is in favour of the mean reversion feature exists. High

interest rates lead to less requirements of fund from borrowers and that forces the interest rates to drop down closely to the long run equilibrium level. In contrast, low interest rates lead to more requirements of fund from borrowers and that forces the interest rates to increase to the long run equilibrium level.

The pricing kernel which controls risk could be given as:

$$m_{t+1} = \log M_{t+1} = -x_t - \frac{1}{2}\lambda^2 - \lambda\varepsilon_{t+1} \quad (2.17)$$

We refer λ as the market price of risk since it determines the covariance between shocks to x and m . The second term in the right hand side is set as $-\frac{1}{2}\lambda^2$ to ensure that the single factor is the short rate.

With the starting condition $\log P_{t+1}^0 = 0$ when $n = 1$, the price of a one-period zero-coupon bond is obtained from the pricing equation (2.14):

$$\begin{aligned} \log P_t^1 &= E_t[m_{t+1} + \log P_{t+1}^0] + \frac{1}{2}\text{var}_t[m_{t+1} + \log P_{t+1}^0] \\ &= E_t[m_{t+1}] + \frac{1}{2}\text{var}_t[m_{t+1}] \\ &= \left(-x_t - \frac{1}{2}\lambda^2\right) + \frac{1}{2}\lambda^2 \\ &= -x_t \end{aligned} \quad (2.18)$$

Combined with equation (2.9), we conclude that:

$$r_t = y_t^1 = -\log P_t^1 = x_t \quad (2.19)$$

This conclusion states that the only underlying state factor in the Vasicek model is the short rate.

The yield of n-period zero-coupon bonds could be derived from the pricing equation (2.14) recursively. We express the logarithm of the prices of n-period zero-coupon

bonds P_t^n as an affine function of the state factor x_t :

$$-\log P_t^n = A_n + B_n x_t \quad (2.20)$$

with the conditions $A_0 = B_0 = 0$, $A_1 = 0$ and $B_1 = 1$. Combining equation (2.20) with (2.17), we obtain this result:

$$\begin{aligned} m_{t+1} + \log P_{t+1}^{n-1} = & - \left[A_{n-1} + \frac{1}{2} \lambda^2 + k\theta B_{n-1} \right] \\ & - [1 + (1 - k)B_{n-1}]x_t - (\lambda + B_{n-1}\sigma)\varepsilon_{t+1} \end{aligned} \quad (2.21)$$

where

$$E_t[m_{t+1} + \log P_{t+1}^{n-1}] = - \left[A_{n-1} + \frac{1}{2} \lambda^2 + k\theta B_{n-1} \right] - [1 + (1 - k)B_{n-1}]x_t \quad (2.22)$$

$$\text{var}_t[m_{t+1} + \log P_{t+1}^{n-1}] = (\lambda + B_{n-1}\sigma)^2 \quad (2.23)$$

Then substitute (2.22) and (2.23) into equation (2.14), the bond price is given as:

$$\begin{aligned} -\log P_t^n = & A_{n-1} + \frac{1}{2} \lambda^2 + k\theta B_{n-1} - \frac{1}{2} (\lambda + B_{n-1}\sigma)^2 \\ & + [1 + (1 - k)B_{n-1}]x_t \end{aligned} \quad (2.24)$$

with

$$A_n = A_{n-1} + \frac{1}{2} \lambda^2 + k\theta B_{n-1} - \frac{1}{2} (\lambda + B_{n-1}\sigma)^2 \quad (2.25)$$

$$B_n = 1 + (1 - k)B_{n-1} \quad (2.26)$$

The yield of n-period zero-coupon bonds could be derived from equation (2.8) with the same condition for A_n and B_n as in equations (2.25) and (2.26):

$$y_t^n = -\frac{1}{n} \log P_t^n$$

$$= \frac{1}{n} \left\{ A_{n-1} + \frac{1}{2} \lambda^2 + k \theta B_{n-1} - \frac{1}{2} (\lambda + B_{n-1} \sigma)^2 + [1 + (1 - k) B_{n-1}] x_t \right\} \quad (2.27)$$

That is the closed-form solution to the Vasicek model and the obtaining of closed-form solution is another essential feature of this model. Although short rate is the only dependent variable for yield curve, the model is still able to construct some different shapes of yield curve occur in reality.

Simple as it is, but we have to admit that the Vasicek model has several shortcomings. Firstly, the yields with all the maturities are perfectly correlated and that is against the yields behaviour in the real world. Secondly, the model is not flexible enough to describe the shapes of the yield curve due to the dependence of the only variable. Finally, the interest rates in Vasicek model have a positive probability to be negative which can't be true since in reality the nominal interest rates could decrease closely to zero but never be negative.

CIR Model

In the CIR (1985) model, the time-varying volatility is included to remove the possibility of observing negative interest rates. That means the conditional variance of the short rate is a constant in the Vasicek model, while it is allowed to change over time in the CIR model. The single state variable x which is the short rate follows a square-root process as:

$$x_{t+1} - x_t = k(\theta - x_t) + \sigma \sqrt{x_t} \varepsilon_{t+1} \quad (2.28)$$

Compared with Vasicek model, the drift is not changed, but the volatility is multiplied with the square-root of the state variable. The drift keeps the property of mean reversion and the modification of volatility ensures the interest rates to be strictly positive. Considering the time-varying volatility, when the interest rates decline closely to zero. When it reaches to zero, this eliminates the random term. With a strictly positive drift term, the interest rates are compelled from zero to positive values.

The pricing kernel equation is set at:

$$m_{t+1} = \log M_{t+1} = -(1 + \frac{1}{2} \frac{\lambda^2}{\sigma^2})x_t - \frac{\lambda}{\sigma} \sqrt{x_t} \varepsilon_{t+1} \quad (2.29)$$

The pricing kernel is conditionally lognormal and the coefficient of x is selected in order to equate the state variable x to the short rate and that could be proved as what we have done for the Vasicek model.

The n-period bond price in CIR model satisfies this equation:

$$\begin{aligned} -\log P_t^n &= A_{n-1} + k\theta B_{n-1} + \left[1 + \frac{1}{2} \lambda^2 + (1-k)B_{n-1}\right] x_t \\ &\quad - \frac{1}{2} (\lambda + B_{n-1}\sigma)^2 x_t \end{aligned} \quad (2.30)$$

with

$$A_n = A_{n-1} + k\theta B_{n-1} \quad (2.31)$$

$$B_n = 1 + \frac{1}{2} \lambda^2 + (1-k)B_{n-1} - \frac{1}{2} (\lambda + B_{n-1}\sigma)^2 \quad (2.32)$$

The yield of n-period zero-coupon bonds could be derived from the equation (II.29) with the same condition for A_n and B_n as in equations (II.52) and (II.53):

$$\begin{aligned} y_t^n &= -\frac{1}{n} \log P_t^n \\ &= \frac{1}{n} \left\{ A_{n-1} + k\theta B_{n-1} + \left[1 + \frac{1}{2} \lambda^2 + (1-k)B_{n-1}\right] x_t - \frac{1}{2} (\lambda + B_{n-1}\sigma)^2 x_t \right\} \end{aligned} \quad (2.33)$$

Multi-factor Affine Models

The single-factor models as presented above are simple and elegant since the yield curve is modelled as a function of just one state variable as the short rate. However, the

drawbacks of short rate models are also caused by its simplicity. The fact that the prices at all maturities are driven by a single stochastic factor implies that the movements of the yield generated by these models are perfectly correlated. This is contradicting with the empirical evidence. Also, the empirical evidence shows that a single-factor model is not sufficient to describe the dynamics of the term structure. For example, Litterman and Scheinkman (1991) discovered that over 98% of the variation in returns on government fixed income securities can be explained by three factors, labelled as level, slope and curvature. Thus, multi-factors models are superior in improving the model fitness and achieving better tractability and flexibility.

The core framework of affine term structure models is constructed by Duffie and Kan (1996), highly recognized by its popularity in analytic tractability. They introduce a generalized affine term structure model which nests a large number of models such as Vasicek (1977), CIR (1985), Hull and White (1990), Longstaff and Schwartz (1992) and so on. A canonical representation of affine models is given by Dai and Singleton (2000) in terms of latent variables which are admissible and maximal. Duffee (2002) proposes the “essentially affine” models which allow the risk compensation to vary independently of interest rate volatilities.

General Affine Model

Duffie and Kan (1996) generalized the affine term structure models which include the Vasicek (1977) and CIR (1985) as special cases in the continuous-time framework and we follow the work of Backus, Foresi, and Telmer (1996) who translate the model into discrete time.

In the discrete-time vision of the Duffie and Kan (1996) model, in order to add more factors to describe the yield curve, the short rate r_t is assumed to be an affine function of n unobserved factors rather than equal to the only state variable in the single-factor model and the n state variables are denoted as an n -vector X_t :

$$r_t = \rho_0 + \rho_1' X_t \quad (2.34)$$

where ρ_0 is a scalar, ρ_1 is an n -vector. Then the n -vector of latent state variables is assumed to follow this process with affine form drift and variance:

$$X_{t+1} - X_t = K\theta - KX_t + \sqrt{S_t}\varepsilon_{t+1} \quad (2.35)$$

where $\theta \in \mathbb{R}^n$, K is an $n \times n$ matrix, $\varepsilon_{t+1} \sim NID(0, I)$ and $V(X)$ is an $n \times n$ diagonal matrix with element

$$S_t^{ii} = \alpha_i + \beta_i' X_t \quad (2.36)$$

with α_i a scalar and β_i a n -vector. Proper restrictions which could be found in Dai and Singleton (2000) are assumed to ensure that $\alpha_i + \beta_i' X_t$ are nonnegative for all i and X_t .

The pricing kernel equation is expressed as:

$$m_{t+1} = \log M_{t+1} = -\rho_0 - \rho_1' X_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1} \quad (2.37)$$

where λ_t is the time-varying market price of risk and $\lambda_t = \lambda \sqrt{S_t}$, λ is a $n \times n$ constant matrix. Bond prices are still assumed to be a log-affine function of the latent factors:

$$-\log P_t^n = A_n + B_n X_t \quad (2.38)$$

By using equation (2.11) with the pricing kernel equation, the recursions give this result:

$$A_n = \rho_0 + A_{n-1} + B_{n-1} K \theta - \frac{1}{2} \sum_{i=1}^k (\lambda_i + B_{ni})^2 \alpha_i \quad (2.39)$$

$$B_n = \rho_1' + B_{n-1} (I - K) - \frac{1}{2} \sum_{i=1}^k (\lambda_i + B_{ni})^2 \beta_i' \quad (2.40)$$

Thus, the n -period yields could be derived as:

$$y_t^n = -\frac{1}{n} \log P_t^n$$

$$\begin{aligned}
&= \frac{1}{n} \{ \rho_0 + A_{n-1} + B'_{n-1} K \theta - \frac{1}{2} \sum_{i=1}^n (\lambda_i + B_{ni})^2 \alpha_i \\
&+ [\rho'_1 + B_{n-1} (I - K) - \frac{1}{2} \sum_{i=1}^n (\lambda_i + B_{ni})^2 \beta'_i] x_{it} \} \quad (2.41)
\end{aligned}$$

Extension of Multi-Factor Affine Models

In order to address some problems of the general affine model introduced by Duffie and Kan (1996), Dai and Singleton (2000) propose the “canonical” representation of affine term structure models.

Firstly, before defining the class of admissible affine term structure of interest rates, they introduce the invariant transformation which consists in making permutation in the state variables and rotation in Brownian motion vector in the way that leaves the implied bond price, short rates and their distributions unchanged.

Then, they give the condition of admissibility for the dynamic equation of the state variables to guarantee the volatility is always positive, since not all of the parameterizations are feasible in the model from Duffie and Kan (1996). That is an important motivation of their canonical representation of affine term structure models which is to treat the drift and diffusion coefficients separately in deriving the conditions for admissibility. It is necessary to impose constraint to the choice of parameter vector to ensure the volatility is strictly positive.

Dai and Singleton (2000) classify each affine term structure models into one of $N + 1$ subfamilies based on the value of m which equals to the degree of dependence of the conditional variances on the number of state variables. They define $\mathbb{A}_m(N)$ subfamilies to be the n -factor affine models which are admissible with index value m . The state variable X_t is partitioned as $X' = (X^{B'}, X^{D'})$ for each m , where X^B is $m \times 1$ and X^D is $(N - m) \times 1$. They give sufficient restrictions on the parameters of $\mathbb{A}_m(N)$ to guarantee the admissibility. And $\mathbb{A}_m(N)$ is the set of all affine term structure models that nested special cases of the canonical model or of any equivalent model obtained by an invariant transformation of the canonical model.

Dai and Singleton (2000) indicates that the canonical representation is also maximal which means that minimal known sufficient conditions for admissibility and minimal normalizations for econometric identification are imposed in $\mathbb{A}_m(N)$ for any given m .

Duffee (2002) estimates a canonical affine model $\mathbb{A}_2(3)$ and indicates that the model could not forecast the future yields. He also shows that the pricing errors in the affine models are strongly related to slope which implies that even a 3 factor affine model with time varying variance fails to capture independent variation in slope. Thus, better specifications of the market price of risk are needed to capture the behaviour of returns in time series. Duffee (2002) proposes the “essentially affine models” which is a generalization of the market price of risk and the key objective is to break the linkage between the expected excess returns to bonds and the volatility of yields.

Two important improvement of essentially affine form are given by Duffee (2002). First, in the canonical affine models, the market prices of risk are assumed to be $\lambda_t = \sqrt{S(t)}\lambda$, while in the essentially affine models, the market price of risk vector is not only determined by the variation in $S(t)$, but also by X^D which means the tight relationship between the expected excess returns and the volatility have been removed and which makes the model much more flexible to fit the empirical behaviour of the expected excess return. Then, the sign of the market price of risk is not allowed to change in the canonical affine model while this restriction is eliminated in the essentially affine model.

Table II-3 No-Arbitrage Affine Factor Models Mentioned in this Chapter

Category	Author	Year	Specification	Description
Single-factor	Vasicek	1977	$x_{t+1} - x_t = k(\theta - x_t) + \sigma \varepsilon_{t+1}$	Simple but the interest rates have a positive probability to be negative

	Cox- Ingersoll- Ross	1985	$x_{t+1} - x_t$ $= k(\theta - x_t)$ $+ \sigma\sqrt{x_t}\varepsilon_{t+1}$	The volatility is multiplied with the square-root of the state variable, which ensures the positive interest rate
Multi- factor	Duffie and Kan	1996	$X_{t+1} - X_t$ $= K\theta - KX_t$ $+ \sqrt{S_t}\varepsilon_{t+1}$ $S_t^{ii} = \alpha_i + \beta_i'X_t$	Generalized affine term structure model
	Dai and Singleton	2000	$X' = (X^{B'}, X^{D'})$	Canonical representation of affine term structure model introduces the invariant transformation
	Duffee	2002	$\lambda_t = \sqrt{S(t)}\lambda$	Essentially affine model removes the tight relationship between the expected excess returns and the volatility. The sign of the market price of risk can change rather than fixed in the canonical affine model.

II.2.3. Macro-Finance Models

As we have introduced in the last section, the “finance term structure models”, such as the canonical affine term structure models due to Dai and Singleton (2000) and the essentially affine form given by Duffee (2002), are all built for nominal bond yields only. And the state variables in these dynamic models are called “latent factors” which

are used to explain the term structure movements, but they have no explicit economic meanings in the real world. For example, Dai and Singleton (2000) label the factors in their model as “level”, “slope” and “butterfly” which describe the factors effects on the yield curve. But the factors are not directly compared with any macroeconomic variables. As to the important role term structure of interest rates plays in macroeconomics, especially on monetary economics. The macroeconomic information contained in term structure of interest rates attracts significantly increased attention from the academic researchers and policy makers, implying it is necessary to encounter the information of macroeconomics to the “nominal yields only” model. Recent literature is trying to add macro variables or theoretical structures to financial term structure models to explore the impact the macroeconomic variables have on yield curve, and also to utilize the information which is contained in term structure of interest rates to the macroeconomic models to improve the estimation efficiency of macro models.

The publication of the celebrated paper of John Taylor (1993) triggered and promoted the developing of the so-called “macro-finance models”. Taylor rule proposed by Taylor (1993) describes how a central bank should adjust its interest rate policy instrument in response to changes in inflation, output, or other macroeconomic activity to foster price stability and full employment. The classical backward-looking Taylor rule expresses the interest rate as an affine function of the inflation rate and output gap. Clarida, Gali & Gertler (2000) extend it to a forward-looking version. According to this type of policy rules, central banks react to expected inflation and expected output gap. Since interest rate in affine factor models is an affine function of the factors, while interest rate is considered as an affine function of some macroeconomic variables under the Taylor rule, macroeconomic variables can be added to the term structure models directly by letting the macro variables under the Taylor rule to be the factors in the finance models. Based on this argument, Ang and Piazzesi (2003) firstly propose the joint dynamics of yields on zero-coupon bonds with macroeconomic variables in a Vector Autoregression (VAR).

Kim (2008) gives the “affine-Gaussian” framework, which is the basic model most

macro-finance models based on in the literature:

$$m_{t+1} = \log M_t = -r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \epsilon_{t+1} \quad (2.42)$$

$$X_{t+1} = \Phi X_t + (I - \Phi)\mu + \Sigma \epsilon_{t+1} \quad (2.43)$$

$$r_t = \rho_0 + \rho' X_t \quad (2.44)$$

$$\lambda_t = \lambda_a + \Lambda_b X_t \quad (2.45)$$

where M_t is the pricing kernel, X_t is an n -dimensional vector of state variables, r_t is the nominal short rate, λ_t is the market price of risk of the n -dimensional shocks ϵ_{t+1} , Φ , Σ , Λ_b are $n \times n$ constant matrices, ρ and λ_a are constant n -dimensional vectors, and ρ_0 is a constant. Equation (2.42) is the pricing kernel in discrete-time form and equation (2.43) is the discrete factors in the affine term structure models, which has a Gaussian (VAR (1)) specification. Equation (2.44) and (2.45) are the affine forms of nominal short rate r_t and market price of risk λ_t respectively.

Those constitute the basic model of “macro-finance” which is called the “affine-Gaussian” model. Most macro-finance models in the literature are built based on this model and the different choices of the restrictions on the matrices such as Φ and ρ leads to the diversity of macro-finance models.

The yield curve is an affine function of factors in the macro-finance models, hence we could view the state variables in the framework as a forming of the so-called “basis” to the term structure of interest rates. Kim (2009) classifies the macro-finance models by the source of the factors comes from and defines as “internal basis” models and “external basis” models. If the state variables are unobservable latent factors which are determined just inside the estimation, it is an internal basis model. For instance, models which are built by using the methods of principle component analysis and Kalman filter are all internal basis models. If the state variables are observable specific macroeconomic variables which are priori fixed completely or partially, it is an external basis model. For example, the model takes inflation, output gap and interest rate as factors is an external basis model. In other words, the difference between the internal

and external basis models is the way they project information where the internal basis models are likely to project information in yields and observable macro variables onto the state vector consisting of unobservable variables. Whereas the external basis models often project information in yield onto observable macro variables and latent variables if there is any.

Internal Basis Models

Generally, the factors in the internal basis models are unobservable latent factors and there is no explicitly economic interpretation of them. The VAR models used to capture the dynamics of the factors have no economic structure. Also restrictions have to be given in order to simplify the estimation procedure during the empirical analysis and usually there is no economic theory to support these restrictions. The restrictions are incorporated flexibly for the simplification of the model estimation. For the models with both unobservable and observable factors, if the factor model is a reduced-form VAR representation of the data with constraint from the economic theory, the macro-finance model will be an internal basis model. In this section, we introduce two popular internal basis models by Ang and Piazzesi (2003) and Diebold, Rudebusch & Aruoba (2006).

Ang and Piazzesi (2003)

Ang and Piazzesi (2003) employ a Gaussian model with both latent yield curve factors and observable macroeconomic variables, inflation and real activity, investigating the one-way influence of macro variables on the dynamics of yield curve. In this model, the no-arbitrage assumption is imposed and the risk premia, which depend on both macro and latent factors, are time-varying. There is no structural macro model behind the model construction and the macro variables are incorporated by using Taylor rule.

Data used in this paper are U.S. monthly zero-coupon yields with maturities 1, 3, 12, 36 and 60 months from 1952 to 2000. Macro data is extracted from two groups of variables separately by first principle component. They are inflation measure group consisting of CPI, PPI and spot market commodity prices, and real activity group

including the index of Help Wanted Advertising in Newspapers, unemployment, the growth rate of employment and the growth rate of industrial production.

The model is estimated by maximum likelihood and the representation of VAR make it convenient to employ the technique of impulse response functions and variance decomposition which allow the author to measure how much of the variation in the yield curve can be explained by the macro factors and latent factors.

Their empirical results indicate that the macroeconomic variables play a more important role in the short end of yield curve than in longer-term maturities. The latent yield curve factors can be interpreted as level, slope and curvature of the yield curve as in previous literature. In addition, imposing no-arbitrage assumption and macroeconomic variables both improves the accuracy of in out-of-sample forecasts.

Diebold, Rudebusch & Aruoba (2006)

Although Ang and Piazzesi (2003)'s work is seminal, the main shortcoming of their model is that the relationship between yield curve and macro variables is assumed as unidirectional rather than bidirectional. The impact of yield curve on macroeconomy cannot be investigated within this framework.

Diebold, Rudebusch & Aruoba (2006) constructed a Nelson-Siegel form macro-finance model to explore the bidirectional interaction between yield curve and macroeconomy. The latent yield curve factors (level, slope and curvature) and observable macroeconomic variables (manufacturing capacity utilization, the federal fund rate and annual price inflation) are connected by using a VAR(1). This model is written in state-space representation, which facilitates estimation and extraction of latent factors by using Kalman filter.

In this paper, U.S. treasury yields from 1972 to 2000 with maturities of 3, 6, 9, 12, 15, 18, 21, 24, 30, 48, 60, 72, 84, 96, 108 and 120 months are examined. The model is estimated in one-step by maximum likelihood via Kalman filter, in which the state equation and transition equation are estimated simultaneously rather than separately in

two steps as in Diebold and Li (2002).

They find bidirectional interactions between the macroeconomy and yield curve, but the macroeconomic effects on the future yield curve is much more significant than the reverse. In addition, they focus their research on the expectation hypothesis and concludes that the expectation hypothesis only holds during certain periods not the whole sample.

External Basis Models

In most of the external basis models, factors are all observable macroeconomic variables and the models usually have explicit economic interpretation. The dynamic factor models are based on the dynamic general equilibrium model from the New Keynesian theory and are estimated by using structural VAR model in the empirical analysis. For the models with both unobservable and observable factors, if the factor model is a structural VAR obtained from some economic theories (e.g. New Keynesian), the macro-finance model will be categorised to external basis models. The macro-finance models from Hordahl, Tristani & Vestin (2006), Rudebusch & Wu (2008) and Bekaert, Cho & Moreno (2010) construct are all external basis models and their dynamic factor models are all based on the New Keynesian economic theory.

Hordahl, Tristani & Vestin (2006)

Ang and Piazzesi (2003) assume that inflation and output are all determined independently of the short-term interest rate. Instead of using a reduced-form VAR of inflation and output, Hordahl, Tristani & Vestin (2006) remove these restrictions by constructing an explicit structural macroeconomic framework rather than a reduced-form VAR representation of the data. The macroeconomy is summarized by a small-scale rational expectations model and the yields are affine functions of the state variables of the macroeconomic model by imposing the arbitrage-free assumption.

Data used in this paper are monthly German treasury yields ranging from 1975 to 1998 with maturities of 1, 3, 6, 12, 36 and 84 months. Macroeconomic data are year-on-year

CPI and output gap.

The empirical results show that the macroeconomic and term structure modelling are complementary. The estimation of the macroeconomic parameters which are partly determined by the term structure data is consistent with the estimation results by using macroeconomic models. Also, the explanatory power of this model for the term structure is similar to that of the term structure models which are constructed only on latent factors.

Rudebusch & Wu (2008)

The second external basis model we introduce is by Rudebusch & Wu (2008). In this paper, they combine a canonical affine no-arbitrage term structure model with a hybrid New Keynesian rational expectations macroeconomic model to investigate the dynamic interactions between yield curve and the macroeconomy. By using a monetary policy reaction function, the short-term interest rate is related to macroeconomic fundamentals. Their model gives a relationship between the no-arbitrage latent term structure factors and the macro variables where the level factor is interpreted as the perceived inflation target and the slope factor is related to the inflation and output gap cyclical variation. The variation on slope factor is considered as a cyclical monetary policy response of the central bank to the economy by changing the short end of the yield curve in order to achieve the macroeconomic policy goals.

They estimate the macro-finance model by using the end-of-month data sampled from January 1988 to December 2000 on the yields of five U.S. Treasury zero-coupon bonds with maturities of 1, 3, 12, 36, and 60 months. The yields are annual rate by using the unsmoothed Fama and Bliss (1987) approach and the output is measured by capacity utilization. The combined macro-finance model is estimated by maximum likelihood method and also compared with a no-arbitrage yield-only term structure model without macroeconomic variables.

Rudebusch and Wu (2008) obtain these main conclusions. Firstly, the latent factors from the affine term structure model seem to have obvious macroeconomic and

monetary policy underpinnings. The level factor is closely related to the perceived medium-term central bank inflation target and the slope factor is related to the cyclical variation on the inflation and output gap which are caused by the movement in the short end of the yield curve controlled by the central bank to achieve the goal of the macroeconomic policy. Secondly, the monetary policy inertia and the slow partial adjustment of the policy interest rate from Federal Reserve are not supported by any empirical evidence. Finally, in the macroeconomic dynamics, the forward-looking and backward-looking are both important.

Table II-4 Macro-Finance Models Mentioned in this Chapter

Category	Author	Year	Specification	Description
Internal-basis	Ang and Piazzesi	2003	Macro variables are incorporated to reduced-form VAR by using Taylor rule	One-way influence of inflation and real activity to yield curve
	Diebold, Rudebusch and Aruoba	2006	Nelson-Siegel form macro-finance model	Bidirectional interactions between yield curve and macro variables, capacity utilization, the federal fund rate and inflation
External-basis	Hordahl, Tristani and Vestin	2006	Constructs an explicit structural macroeconomic framework	A small-scale rational expectations model summarises the macroeconomy

	Rudebusch and Wu	2008	Combines canonical affine term structure model with a hybrid New Keynesian rational expectations macroeconomic model	The short-term interest rate is related to macroeconomic fundamentals by using monetary policy reaction function
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II.2.4. Markov Regime-Switching Models

The short-term interest rate plays a key role in the valuation of almost all securities. In the affine factor models, the short-term interest rate which represents the instantaneous interest rate is the only or one of the state variables that determine the entire movements of the yield curve. Thus, the fundamental role of short-term interest rate makes it the most frequently modelled variable in financial economics. Many studies support the existence of regime changes in the dynamics of short-term interest rate. Volatility in U.S. short rate is found higher during certain episodes with financial crisis, abrupt government policy changes or other events. The short rate behaves quite differently under various economic environments. Therefore, modelling the time changing behaviour of short rate explicitly can provide better fitting to data than homogeneous model.

The Markov regime-switching models introduced by Hamilton (1989), employs the switching variable which follows a Markov chain process to capture the various behaviour of a time series within each regime. Applications to the short-term interest rate modelling includes Gray (1996), Ang and Bekaert (1998) and Kalimipalli and Susmel (2001) and so on.

The Markov regime-switching model proposed by Hamilton (1989) describes the evolution of an economic variable by a first-order autoregressive model,

$$y_t = c_1 + \phi_1 y_{t-1} + \varepsilon_t \quad (2.46)$$

with $\varepsilon_t \sim N(0, \sigma^2)$. Since we would like to capture the behaviour of the variable under various state of economy, a different set of parameters which interprets the new evolution of the variable is in demand. Thus, the description of the data according to the episodes showing diverse behaviour from equation (2.46) could be captured as

$$y_t = c_2 + \phi_2 y_{t-1} + \varepsilon_t \quad (2.47)$$

Equation (2.46) and (2.47) can be combined into on notation,

$$y_t = c_{s_t} + \phi_{s_t} y_{t-1} + \varepsilon_t \quad (2.48)$$

where s_t is the unobservable regime indicator. Assume there is only two regimes in this model, the regime indicator is normally assumed to be 1 when the variable behaves in the first regime (such as in booming) and 2 when it behaves differently in another regime (such as in recession).

The regime indicator s_t is assumed to follow a two-state (for example of two regimes) Markov chain with

$$p_{ij} = \Pr(s_t = j | s_{t-1} = i, s_{t-2} = l, \dots, y_{t-1}, y_{t-2}, \dots) = \Pr(s_t = j | s_{t-1} = i) \quad (2.49)$$

This is the basic Markov regime-switching model which allows the regimes to be shifted randomly in any order and any times. The transition probabilities assumption and the maximum likelihood estimation details is given in chapter 5. The empirical studies applying the Markov regime-switching into the modelling of short-term interest rate are reviewed as below.

Gray (1996)

Gray (1996) constructs a generalized regime-switching (GRS) model of short rate which is flexible enough to model the short rate data generated within various economic

mechanisms. The transition probabilities which governs the regime shift is assumed to be state-dependent rather than fixed. In addition, Gray (1996) nests the generalized autoregressive conditional variance (GARCH) effect and CIR specification to this model. The empirical result indicates that the GRS model provides best performance in modelling the stochastic volatility of short rate. The U.S. is found behaving as a random walk in the regime with low volatility and tending to reverse to a long-run mean in high volatility regime.

Ang and Beaert (1998)

The empirical study from Ang and Beaert (1998) compares the regime-switching model performance of short-rate with data from the U.S., Germany and the UK. They find strong evidence of the existence of regime shifting. However, Ang and Beaert (1998) claims that one cannot receive consistent estimation of parameters by using univariate models. Therefore, they suggest to incorporate international short rate and term spread information to improve the regime-switching model.

Kalimipalli and Susmel (2001)

Kalimipalli and Susmel (2001) introduces regime-switching in a two-factor model which nests level and stochastic volatility effects. In their model, the volatility follows a stochastic volatility process and the mean of it subject to shifts within different regimes. The model is estimated by using Gibbs Sampling based Markov Chain Monte Carlo algorithm. The empirical results indicate that the two-factor regime-switching stochastic volatility model outperforms the other two-factor models both in in-sample fitting and out-of-sample forecasting.

II.3. Background of Chinese Government Bond Market

In this section, the background of Chinese government bond market is presented. The development history of the Chinese government bond market is reviewed in the first section. And the overview incorporates structure and functioning of the market is described in the second section. Primary and secondary markets are introduced

respectively from various aspects including issuance and trading volume, maturity composition, segmentation of sub-secondary markets, market participants pattern and so on. In section 3, the monetary policy conducted in China is presented briefly and the achievements which has been made in the interest rates liberalisation is described in the following part. In the last section, major problems of Chinese government bond market are pointed out.

II.3.1. Review of History

China's government debt history could be traced back to the end of 19th century. The first domestic loan in China was issued by the Qing government in 1894 for the purpose of raising money for the Sino-Japanese war. In the following periods, the Qing government issued another two domestic loans, the Northern Warlord government issued 27 and the Republic government issued 86 domestic loans as well.

Since the foundation of the People's Republic of China, the People's Victory Parity Bond was firstly issued by the Ministry of Finance in 1950 for the recovery of the new born country. Later from 1954 to 1958, the State Economic Construction Bond which was issued 5 times to accelerate the economic development under the traditional system, forces the government bonds to be issued only to the state-owned enterprises and public institutions. In the following 20 years, no government bond was issued and the issuance was resumed in 1981 in consequence of economic reform. At that time, no secondary market existed in China, since the government bond issuance was done through administrative allotment by government and trading or transferring of bond was prohibited. The secondary market was introduced to 61 selected cities in 1988 and extended to the whole country in 1990. Over-the-counter (hereafter OTC) was the only platform for trading during this period.

At the end of 1990, stock exchange markets were open both in Shanghai and Shenzhen. The government bond future's trading open in the stock exchange market, but closed in 1995. Subsequently the government closed the OTC market which left the stock exchange to be the only secondary market at that moment. In December 1996, a long-distance auction system was established with the China Central Depository & Clearing

Corporation which is a centralized securities depository. All government bonds were issued through this bond custody and settlement system since then.

In order to isolate the Chinese banking system from market risks associated with exchange transactions, the People's Bank of China pulled commercial banks out from the stock exchange markets and opened the inter-bank market in 1997. The participants of the inter-bank market extended from 16 commercial banks in 1997 to 880 institutions in 2002 with additions of insurance companies, fund management companies, securities firms, leasing companies and so on. And the OTC market was reopened recent years, but it only accounts for a small share of activity.

In April 2001, in order to raise the liquidity, a market maker was introduced and investors could trade or transfer among different markets. Also, in 2005 the book-entry treasury bonds were firstly issued in both stock exchanges and inter-bank markets at the same time. Since then the book-entry treasury bonds could be issued in two or all the three markets simultaneously.

II.3.2. Overview of Chinese Government Bond Market

Nowadays Chinese government bond consists of two types which are book-entry treasury bonds and saving bonds. Book-entry treasury bonds are mostly issued and traded in the stock exchange and inter-bank markets for institutional investors, while the saving bonds are only sold at the bank counters for individual investors and can't be traded, but could be redeemed before maturity or used as collateral. The majority of government bond markets is taken by the book-entry treasury bonds. In the whole year of 2014, 70.94% of new issued government bonds are book-entry treasury bonds and 93.52% of outstanding government bonds are book-entry treasury bonds until the end of 2014.

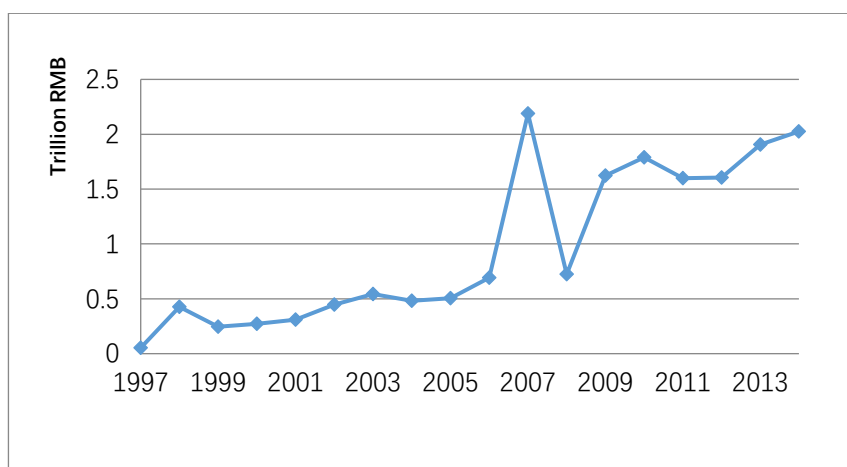
The Ministry of Finance issues government bonds in China. Generally, the Ministry of Finance releases the issuance plan of 1-, 3-, 5-, 7- and 10-year bonds (named key term bonds) at the end of the previous year and announces the release planning of next quarter at the end of each quarter.

Saving bonds are underwritten by syndication and sold to the individual investors via settled bank counters on commission basis once the dealing proportion and amount has been confirmed with the Ministry of Finance. The dealer group is adjusted every three years and consists of 38 commercial banks from 2012 to 2014.

Book-entry treasury bonds are issued through the long-distance auction system to primary dealers. The issuance is market-oriented by competitive interest rates bidding according to the market demand and supply. The dealers are commercial banks, security firms, insurance companies, trust and investment corporations and some other financial agencies. The Ministry of Finance is responsible for the qualification check and approval of the qualified dealers which includes 55 agencies in 2014.

As shown in Figure II-1, Chinese government issuance volume increased remarkably from 2005 and stayed above 1.5 trillion RMB since 2009. In 2014, 2.02 trillion RMB of government bonds were issued which is more than 4 times and 38 times of those issued in 2005 and 1997 respectively. The issuance volume reached a new high record in 2007 at 2.3 trillion RMB due to the launch of 1.55 trillion RMB special treasury bonds which was used to purchase \$20 billion foreign exchanges as the setting-up capital of China's foreign exchange investment company. Chinese special treasury bonds have been issued twice in the history and the other issuance of 0.27 trillion RMB was in purpose of financing wholly state-owned commercial banks in August 1998.

Figure II-1: Issuance volume of Chinese government bond market



Source: ChinaBond

Government bonds in China are issued with various maturities ranging from 3 months up to 50 years. However, before 1996, majority of the issuance were medium term bonds with 3-, 7- and 10- year maturities. Short term bills and long term bonds were added in for diversification since the long-distance auction system was set up. 2-year bonds were added in 1997 and then 5- and 30-year bonds were introduced in 1998. The following additions are the 8-year bonds (1999), 1-year bonds (2000), and 15- and 20-year bonds (2001). Bonds with maturities less than 1 year are all issued after 2003 and the first 50-year super-term bond was launched in November 2009 in order to meet demand from pension funds and insurance companies.

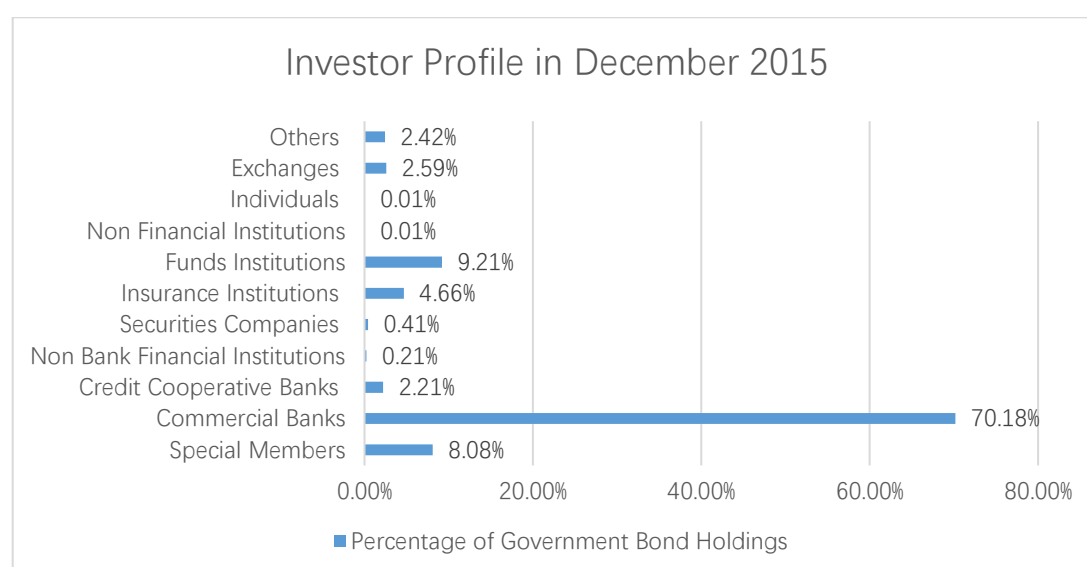
China has three segmented sub-secondary bond markets which are the inter-bank market, stock exchange market (in Shanghai and Shenzhen) and the OTC market. The inter-bank market is quote-driven and governed by the People's Bank of China while the stock exchange market is order-driven and under the conduction of China Securities Regulatory Commission. The OTC market is an essential supplement to the other two markets. As we have introduced, these three sub-secondary bond markets played different roles in various time periods. In December 1997, when the inter-bank market was established, the depository holding in stock exchange market accounted for approximate half of the outstanding government bonds, while inter-bank was responsible for 30% and the other for 20% (OTC was closed at that time). In December 2011, as to the expansion of inter-bank market for years, the stock exchange market depository holding decreased dramatically to 3%, while the inter-bank market increased rapidly to about 93% and OTC only corresponded to 4% of the outstanding government bonds. The inter-bank market absorbed 87.44% of total government bond trading volume in 2014 and the OTC market accounts for 11.6%. Only less than 1% government bond trading occurred in the stock exchange markets.

In the secondary market, government bonds are traded in the form of repurchasing and cash transactions both in inter-bank and stock exchange markets and the repurchasing volume is much bigger than the cash bonds. In 2012, the government bond transactions in cash was 74.38 trillion RMB while the repurchase agreement trading volume was

178.55 trillion RMB which accounts for 70.59%. The cash bonds volume increased by -0.6% and 17.6% in 2010 and 2011 while the repurchasing grows by 27.3% and 48.9%. The statistics indicates that the repurchasing volume of government bonds rises much faster than cash bonds. Also most of the cash and repurchasing transactions occurs in the inter-bank market, especially for the cash bonds. In 2012, the trading volume of cash bonds in the inter-bank market is 73.79 trillion RMB accounting for 99.21% of total cash bond trading and the repurchasing trading approaches 141.7 trillion RMB in inter-bank market which takes 79.36% of whole repurchasing transactions in the secondary markets. Furthermore, 19.4% repurchasing occurred in the Shanghai stock exchange and Shenzhen stock exchange only took a very small share of market.

A large proportion of the investors participated in the Chinese government bonds market are commercial banks. As displayed in Figure II-2, at the end of 2015, commercial banks were responsible for 70.18% of the outstanding government bonds and the second larger proportion 9.21% is accounted by funds institutions. The special members including People's Bank of China, the Ministry of Finance, Policy Banks held 8.08% of the total outstanding. The remaining proportion were taken by insurance institutions (4.66%), credit cooperative banks (2.21%), non-bank financial institutions (0.21%), securities companies (0.41%), exchanges (2.59%), individuals (0.01%) and other investors (2.42%).

Figure II-2: Investor Profile in December 2015



Source: ChinaBond

II.3.3. Monetary Policy in China

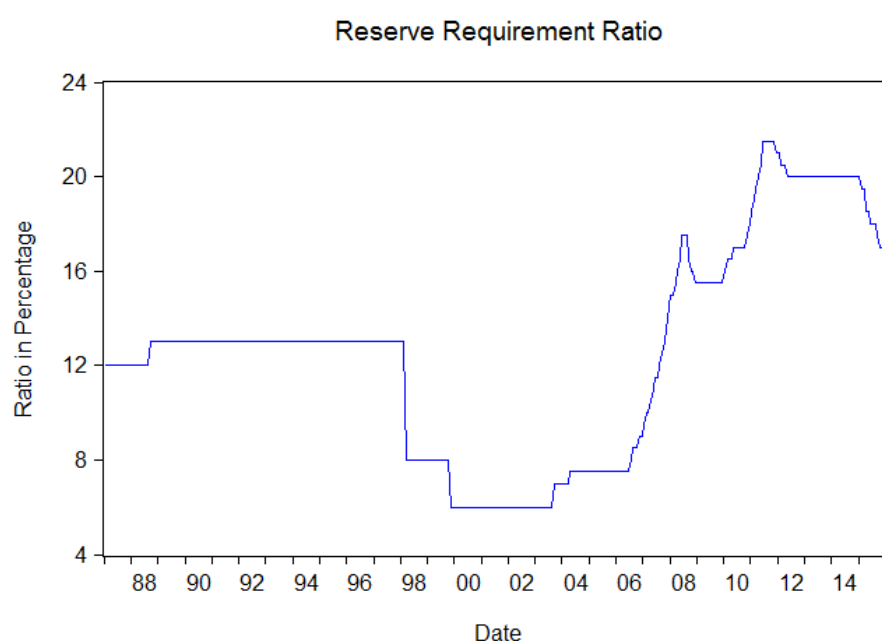
The objective of monetary policy in China is stated as to maintain the stability of the value of the currency and thereby promote the economic growth by the People's Bank of China (PBC). The PBC also undertakes the responsibility of achieving stable exchange rate and job creation. The broad monetary aggregate (M2) is set as the intermediate and operating target. The PBC employs a variety of policy instruments which are open market operations, reserve requirements, central bank lending, standing lending facility, central bank base interest rate and other policy instrument specified by the State Council.

The open market operation is a monetary policy frequently used by advanced economy. The central banks buy or sell government securities in the open market in order to supply liquidity to commercial banks or take the surplus liquidity and control the total money supply indirectly. China's open market operations includes both RMB and foreign exchange operation. Trading securities between the PBC and commercial banks include cash transaction, repurchasing transaction (includes reverse repurchasing) and central bank bill which is the short-term government bond. In January 2013, based on

the worldwide existing monetary policy operation framework and experiences, the PBC launched the Short-term Liquidity Operations (SLO), as a necessary complement to regular open market operations. The main tools of SLO are repurchasing agreements and reverse repurchasing contracts with maturity of less than seven days.

The reserve requirement ratio (RR) is a fraction of deposits that commercial banks must hold as reserves. The liquidity could be drained or injected into the banking system by changing the RR. As shown in Figure II-3, the reserve requirements have been operated extensively as a monetary policy tool since 2006. Significant increasing on RR can be observed during the periods 2007-2008 and 2010-2011 and a sharp decline occurs since 2015. The PBC raised RR ten times in 2007 and eleven times in 2010 and 2011 to manage the excess liquidity and tame inflation. Since 2015, the central bank cut the RR to increase money supply and stem the slowdown speed of the economy.

Figure II-3: Evolution of Reserve Requirement Ratio in China from 1987 to 2016



Source: PBC

The rate at which the Central bank lend loans to financial institutions is referred to as

rediscount rate. By increasing or reducing this rate the PBC makes it more or less expensive for banks to borrow from the central bank.

The PBC launched the Standing Lending Facility in 2003 to smooth out liquidity fluctuations and influence capital costs. This tool allows banks to borrow from the PBC with maturities from one to three months. The flexibility of the Standing Lending Facility is that the borrowing interest rates is determined on a case-to-case basis. It enables the central bank to target banks without affecting the others. Most central banks use the similar tools with different names, such as the Fed's Discount Window, the European Central Bank's Marginal Lending Facility, Bank of England's Operational Standing Facility, the Bank of Japan's Complementary Lending Facility and so on.

The one-year lending and one-year deposit rates are considered as the benchmark rates administered by the PBC in China for direct interest rates control. Since 2003, the interest rates liberalization has made prominent progress. Up to the end of 2015, the situation is that the bank's lending rates have been fully liberalized in 2013 and the baseline deposit rate constitutes a ceiling for deposit rates which is 1.5 times of the official benchmark deposit rate.

II.3.4. Achievements of Chinese Interest Rate Liberalization

Since 1978, China has established the economy restructure plan which aims to transfer from the government-controlled so called planned economic system to a form of socialist market system with Chinese characteristics. After near 40 years of development, China has become the second largest economy in the world and remarkable success has been achieved in various aspects. The financial reform is widely considered as the kernel of the economic shift and is also in purpose of building a domestically driven and consumption-based economy instead of the current investment-driven system. One of the financial reforms and the most indispensable and essential one in recent years is the interest rate liberalization. The government and the PBC is committed to minimize the disruptions of the interest liberalization to the stability of the economy with an appropriate pace, timing and sequence. The efforts have been done for the liberalization in China until the end of 2015 is shown in Table

II-5.

Table II-5: Achievements of Interest Rates Liberalization in China

Year	Achievements
1996	Liberalized interbank offered rates Market-oriented government bond issuance in stock exchange
1997	Liberalized interbank bond repo rates and spot rate
1998	Liberalized policy financial bond rates (CDB ² and EXIMCH ³ issuance) The PBC ⁴ introduces a floating range of lending rates Market-oriented government bond issuance in interbank market
1999	Floating range of lending rates is set at 0.9x, 1.1x for large enterprises and 0.9x, 1.3x for small and medium-sized enterprises
2000	Controls of foreign exchange deposit rates for large accounts and all foreign exchange lending rates are removed
2003	Gradually abolished controls on foreign currency deposit and lending rates in domestic market Upper and lower limits for small amount foreign exchange deposits rate with maturity longer than 1 year are removed
2004	Removed ceiling on lending rates of financial institutions, lowered floor to 0.9x benchmark rates Removed floor on deposit rates of financial institutions
2006	Lower limit of mortgage loan rate is set at 0.85x benchmark
2008	Lower limit of mortgage loan rate is lowered to 0.7x benchmark
2012	Raised deposit rate ceiling to 1.1x benchmark rates Lowered lending rate floor to 0.7x of benchmark rates Lending rates were fully liberalized
2013	Allow nine commercial banks to submit the lending rate they charge their best quality clients each day to set the prime rate Allow the issuance of large negotiable certificates of deposit on the interbank market
2014	Increased deposit rates ceiling to 1.2x Liberalized 5-year and longer deposit benchmark rates
2015	Increased deposit rates ceiling to 1.5x benchmark rate

Source: PBC

As given in the timeline above, the first step China took for the interest rate liberalization reform was in 1996 when the interbank lending market is fully liberalized

² China Development Bank.

³ Export-Import Bank of China.

⁴ People's Bank of China.

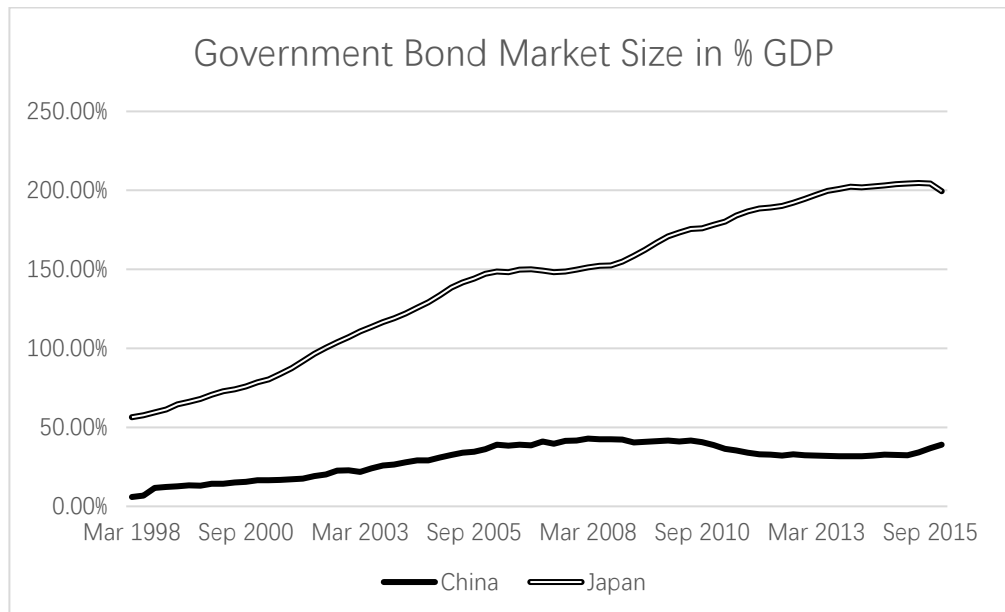
and by 1999 government bond and financial institution bond prices are all decided by the market. In 2001, China joined the World Trade Organization and promised to open up Chinese capital market in the next five years. Under the pressure of this commitment, ceiling on lending rates and floor on deposit rates for financial institutions are all abolished in 2004 and banks are authorized to price the customer risk within a floating range with lower limits on lending rates and upper limits on deposit rates. Then after a decrease of the floor of lending rates in 2012, the lending rates were fully market-oriented with the eliminating of floor on July 2013. The implementation of these deregulation gives more autonomy to commercial banks and also promotes the role of interest rates on optimizing the allocation of financial resources and regulating the macroeconomic operation.

II.3.5. Problems in Chinese Government Bond Market

Since the resumption in 1981, the Chinese government bond market has been developed for more than 30 years. The market size increases rapidly and the interest rates have been liberalized progressively. However, compared with advanced economy, Chinese government bond market still needs further development in many aspects. Some major problems of the Chinese government bond market have been pointed out.

Firstly, although the size of Chinese government bond market is large in par, but in percentage of GDP is small. In Figure II-4, the government bond market size in percentage of nominal GDP in China keeps below 50% while the percentage in Japan increased dramatically from 50% in 1998 to 200% in 2015.

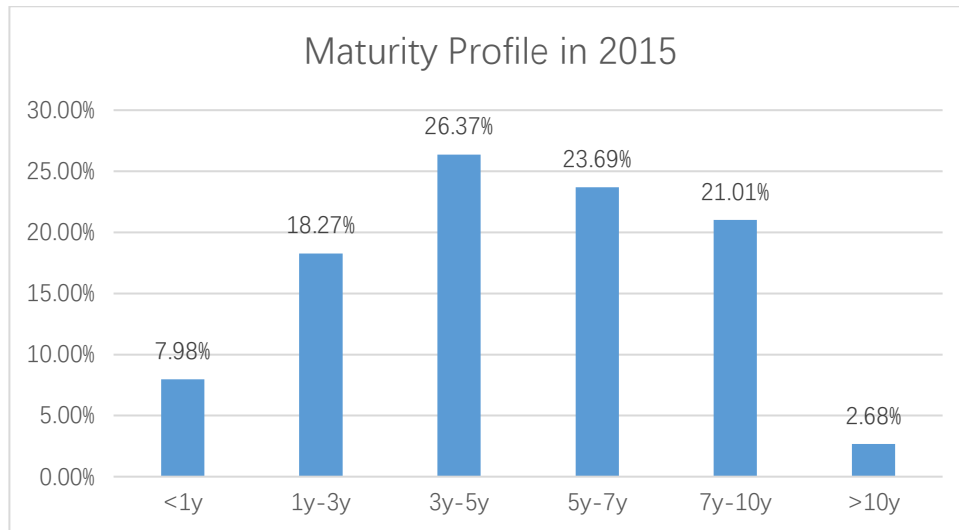
Figure II-4: Chinese and Japanese Government Bond Market Size in Percentage of GDP



Data Source: ChinaBond, Japan Securities Dealers Association, CEIC Data Company Ltd and Bloomberg.

In addition, the lack of short-term and long-term treasury bonds is another problem in Chinese government bond market. As shown in Figure II-5, in 2015 the issuance of government bonds with maturities less than 1 year only accounts for 7.98% and long-term bonds above 10 years takes 2.68 percent. By contrast, the remaining almost 90% of the government bonds were issued as medium term bonds with maturities between 1 and 10 years.

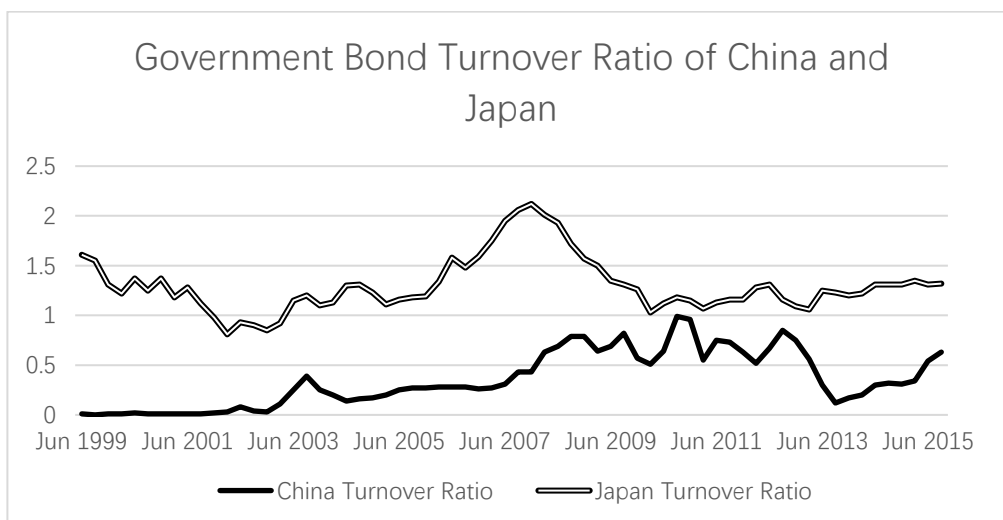
Figure II-5: Maturity Profile of Chinese Government Bond Issued in 2015



Data Source: ChinaBond

Furthermore, the liquidity of Chinese government bond market is relatively poor. The bonds liquidity is normally measured by turnover ratio which is the proportion of trading in the secondary markets over the total outstanding issues. As shown in Figure II-6, the turnover ratio of Chinese government bonds increased from 2003 and stays below 1 until September 2015. By contrast, in Japan the turnover ratio fluctuates around 1.5 and keeps above 1 most of the time.

Figure II-6: Government Bond Turnover Ratio of China and Japan



Data source: ChinaBond and Japan Securities Dealers Association.

At last, although the Chinese interest rates are becoming market-oriented increasingly, there is still a long way to achieve interest rate liberalization. The interest rates are still under administrative forces comparing with fully complete market worldwide.

II.4. Literature in China

As given a strand of worldwide literature review on term structure of interest rate and the current condition of Chinese government bond market, a brief review of literatures on term structure of interest rates in China is presented in this section.

II.4.1. Yield Curve Fitting

Zheng and Lin (2003) employ both bootstrap and polynomial spline methods to approximate Chinese term structure of interest rates. They follow the work of Fama and Bills (1987) and McCulloch (1971) respectively and compare the results from each method. The data set they use is the government bond price on 13th September, 2002 collected from Shanghai stock exchange. Their results show that Chinese yield curve is upward sloping with average of 2% at the short end and 3.5% around the long-end.

In contrast, Zhu and Chen (2003) fits the yield curve by using three order polynomial spline and Svensson (1994) models. They examined 15 government coupon-bond prices on 28th March, 2003 to extract the market rates and concludes that the Svensson (1994) is much more reasonable than polynomial spline in estimating Chinese yield curve since it avoids the overfitting issue at the long end of yield curve.

Guo and Li (2007) achieved similar results. They estimated the yield curve on 31st May, 2006 with 38 coupon-bearing government bonds prices at maturities less than one year, by using cubic spline and Nelson-Siegel (1987) methods. The in-sample fitting and out-of-sample forecasting within the period from July 2005 to June 2006 is compared. They find that the Nelson-Siegel (1987) model outperforms the cubic spline both in-sample and out-of-sample at maturities longer than 3 years and conclude that the Nelson-Siegel

is much more appropriate for emerging market without enough liquidity. In contrast, Bliss (1997) and Ioannides (2003) find empirical evidence from advanced market on that cubic spline method provides better in-sample fitting while Nelson-Siegel outperforms in out-of-sample forecast.

Other studies on yield curve fitting include Yang and Cao (2002), Wang and Wang (2004), Fu and Jiang (2005) and so on. These researches all concentrate on the model selection and comparison for yield curve fitting and the Nelson-Siegel framework is widely considered to be the best model for China due to its simplicity and stability on the long end of yield curve.

II.4.2. Dynamic Modelling

Comparing with the large amount of literatures worldwide on the modelling of term structure of interest rates, studies on the dynamics of the whole yield curve in China is rare. In China the existing researches mainly concentrate on the yield curve fitting which have been discussed in the last section, and the dynamic modelling of short-term interest rate rather than the overall yield curve.

The pioneer work by Luo, Han and Zhang (2012) investigates the dynamics of Chinese yield curve by using three dynamic Nelson-Siegel class models and the predictability of yields at 1-, 5-, 21-, 63- and 126-day horizons is analysed. The data set in their research is daily Chinese inter-bank treasury bond yields from March 2006 to April 2009 at maturities of 6 months, 1, 2, 3, 5, 7, 10, 15, 20 and 30 years. They find that all the three models fit the data well and the more flexible models achieve better in-sample fitting. As to forecasts, they suggest that different specifications should be placed for forecasts at different horizons.

The other related work is limited which is due to the late establishment of Chinese government bond market. And the market had been highly regulated and the trading was not active especially on short-term and long-term bonds for a long time. In contrast, the researches on the volatility of short rate is much more fruitful. The 7-day repo rate was widely considered as the proxy for market short rate before 2006 and after that both

7-day SHIBOR and one-month inter-bank treasury yield are also examined.

Xie and Wu (2002) estimate Vasicek and CIR models by using General Method of Moments. They employ the inter-bank one-month yield as the instantaneous rate and find that the Vasicek model shows better fitting to the market data than the CIR model.

Hong, Lin and Wang (2010) employs four groups of short-term interest rate models including single-factor diffusion, GARCH, Markov regime-switching and jump-diffusion models to explore the dynamic behaviour of Chinese spot rate. Daily data of 7-day repo rate ranging from 1997 to 2008 is used. They find that some important features of Chinese spot rate can be captured by incorporating GARCH, regime shifting and jump effect, but all the considered models are rejected. They also argue that Chinese short rate is significantly influenced by institutional changes, interest rate policy changes and stock market IPOs.

Similar studies include Pan and Shao (2004), Liu and Zheng (2006), Yu and Wang (2008), Zhang and Zhou (2008), Fan (2010) and so on. In all, all these studies on the dynamics of short rate have reached a consensus from three aspects. Firstly, Chinese short rate shows a mean-reversion tendency with fat tails and volatility clustering. In addition, Vasicek model is empirically considered to provide better fitting to data than CIR model. Furthermore, incorporating regime shifting and jump to diffusion model can improve the fitness of degree.

Chapter III. Chinese Government Yield Curve Analysis with Cyclical Mean

III.1. Introduction and Motivation

One of the fundamental features of interest rates is the cyclical behaviour which has been discussed in Kessel (1971), Friedman (1986), Roma and Torous (1997), among others. According to the literature from advanced economies, the specific interest rates cycle is related to the business cycle with increasing of interest rates at business expansions and decreasing at contractions. We are interested in examining that if the assumption of the cyclical tendency of interest rate could help explain the whole yield curve in China.

A large amount of one-factor term structure of interest rates models has been proposed by previous studies. Such as Vasicek (1977), Cox, Ingersoll, and Ross (1985, CIR hereafter) and Brennan and Schwartz (1980). Litterman and Scheinkman (1991) and Chapman David (2001) both claim that almost all the variation of interest rates can be explained by the first three principle components interpreted as level, slope and curvature of yield curve. However, the first factor is necessarily enough to capture more than 90% of the movement on yield curve and the short rate of interest rates is commonly identified as the only state variable in one-factor term structure models. Three reasons for the convenience of using short rate as the only factor in term structure model are given. Firstly, it averts the awkward challenges from elaborating the linkage between the short rate and the other state variables. Secondly, all the interest rate contingent claims are only influenced by the short rate and time. Lastly, closed-form expressions of bond prices and interest rate derivatives prices can be easily derived.

Several studies have explored the Chinese term structure of interest rates based on one-factor short-rate models. Xie and Wu (2002) estimate Vasicek and CIR models by using General Method of Moments. They employ the inter-bank one-month yield as the instantaneous rate and find that the Vasicek model shows better fitting to the market data than the CIR model. Lin and Zheng (2005) also compare the Vasicek and CIR

model by using weekly data from 2001 to 2003 on Chinese yield curve and the result is consistent with Xie and Wu (2002)'s finding. The simpler Vasicek outperforms the CIR on Chinese government bond market. These empirical studies all confirm that the Vasicek model could provide good in-sample fitting to Chinese yield curve.

Therefore, we allow the Chinese interest rates to move cyclically and introduce an extension of Vasicek model which combines the cyclical movements effect of interest rates with the one-factor term structure model to Chinese government bond market. Following the work from Moreno, Novales and Platania (2013), the constant long-run equilibrium level set in the Vasicek model is replaced by a time-varying Fourier series in order to capture the cyclical factor in the fluctuation of interest rates. The Fourier series provides an approximation of an arbitrary periodic function by decomposing it into the sum of simple oscillating functions. This feature not only facilitates the capture of periodic movement of interest rates, but also allows for great flexibility of fitting to the yield curve with various shapes and high analytical tractability of the model estimation. Moreno, Novales and Platania (2013) shows that the Fourier extension model outperforms both Vasicek and the Nelson-Siegel model in in-sample fitting and out-of-sample forecasting by using the U.S treasury data. We are interested in the question that if this model could provide a better estimation on the Chinese yield curve when the Fourier series is incorporated and to what extent. To the best of our knowledge, this study is a pioneer work which brings in the cyclical effect to the modelling of Chinese term structure of interest rates.

This paper is organized as follows. Section 2 presents model construction, estimation methods and prediction approach. Section 3 provides the empirical analysis of in-sample fitting of the model based on two sample periods. The whole period includes the 2008 financial crisis while the post crisis period starts from 2009. Also the out-of-sample forecasting is given for three sub-periods with various yield curve shapes to explore the out-of-sample forecasting power of this model under different economic conditions. The last section is the summary and conclusion of this research.

III.2. Fourier Models and Methodologies

Following the work from Moreno, Novales and Platania (2013), the Fourier model with one term in the Fourier series is presented. Based on their work, we extend the one term Fourier model to a two-term Fourier model by keeping two terms of the Fourier series in the reversion mean.

III.2.1. The One Term Fourier Model

The cyclical mean model is a continuous-time term structure of interest rates model which is based on the conventional Vasicek (1977) model. In the Vasicek model, the instantaneous interest rate is assumed to converge to a long run equilibrium constant value, while in this cyclical mean reversion model, the constant value is changed to be a cyclical long-term level which is described as a Fourier series.

The cyclical mean reversion model specifies that the instantaneous interest rate denoted by r_t follows the Ornstein-Uhlenbeck process which is expressed by the stochastic differential equation as below:

$$dr_t = \kappa(f(t) - r_t)dt + \sigma dW_t \quad (4.1)$$

where $\kappa, \sigma \in \mathbb{R}^+$ and W_t is a standard Wiener process. Since only the real part of the Fourier series has an economic meaning, only real part is considered in this model. Also, the cyclical mean reversion level denoted by $f(t)$ is assumed to follow a Fourier series

$$f(t) = \sum_{n=0}^{\infty} \text{Re}[A_n e^{in\omega t}] \quad (4.2)$$

where $\forall n \mid A_n \in \mathbb{C}$. The phase factor contained in A_n could be defined as $A_n = A_{n,x} + iA_{n,y}$ where $A_{n,x}, A_{n,y} \in \mathbb{R}$. $A_{n,x}$ is the amplitude of the instantaneous rate fluctuations and the $A_{n,y}$ is the phase.

Under the risk-neutral measure \widetilde{P} , the standard Wiener process is expressed as $\widetilde{W}_t =$

$W_t + \lambda t$, where the market price of risk $\Lambda(r_t, t)$ is a constant which equals to λ . Then the risk-neutral vision of the SDE in (4.1) could be given as below:

$$dr_t = \mu_r dt + \sigma d\widetilde{W}_t \quad (4.3)$$

where

$$\mu_r = \kappa(\alpha + g(t) - r_t) \quad (4.4)$$

$$\alpha = A_0 - \frac{\lambda\sigma}{\kappa} \quad (4.5)$$

$$g(t) = \sum_{n=1}^{\infty} \text{Re}[A_n e^{in\omega t}] = f(t) - A_0 \quad (4.6)$$

By using the Itô's lemma, no-arbitrage constraint and probabilistic techniques, the price of a zero-coupon bond at time t with maturity T and par value £1 is expressed as

$$P(r_t, t, T) = e^{A(t, T) - B(t, T)r_t} \quad (4.7)$$

where

$$\begin{aligned} A(t, T) = & \frac{\sigma^2}{2\kappa^2} \left[(T - t) - 2B(t, T) + \frac{1 - e^{-2\kappa(T-t)}}{2\kappa} \right] + (B(t, T) - (T - t))\alpha - \\ & \sum_{n=1}^{\infty} \text{Re} \left[\frac{A_n}{n\omega(\kappa + in\omega)} (e^{in\omega t} (n\omega e^{-\kappa(T-t)} + i\kappa - n\omega) - i\kappa e^{in\omega T}) \right] \end{aligned} \quad (4.8)$$

$$B(t, T) = \frac{1 - e^{-\kappa(T-t)}}{\kappa} \quad (4.9)$$

Since the yield to maturity $R(r_t, t, T)$ could be given in form of bond price $P(r_t, t, T)$ as follows,

$$R(r_t, t, T) = -\frac{1}{\tau} \ln P(r_t, t, T), \tau = T - t \quad (4.10)$$

we plug in the expression of bond price in (4.7) and keep only the first term of the Fourier series for simplicity, then the in-sample fitting model could be given for each

maturity j as

$$Y_{j,t} = \delta_1 z_{1j,t} + \delta_2 z_{2j,t} + \delta_3 z_{3j,t} + \delta_4 z_{4j,t} + u_{j,t} \quad (4.11)$$

where:

$$Y_{j,t} = R(r_t, t, T) - \frac{B(t, T)}{T - t} r_t$$

$$z_{1j,t} = \frac{B(t, T)}{T - t} - 1$$

$$z_{2j,t} = \frac{1}{2\kappa^2} - \frac{B(t, T)}{(T - t)\kappa^2} + \frac{1 - e^{-2\kappa(T-t)}}{4(T - t)\kappa^3}$$

with $B(t, T) = \frac{1 - e^{-\kappa(T-t)}}{\kappa}$, $\delta_1 = \alpha$, $\delta_2 = \sigma^2$, $\delta_3 = A_x$ and $\delta_4 = A_y$. And $u_{j,t}$ is the error term.

The first term of the Fourier series we have taken could be given in this form:

$$\text{Re}[-(A_x + iA_y) \left(\frac{e^{i\omega t} (\omega e^{-\kappa(T-t)} + i\kappa - \omega) - i\kappa e^{i\omega T}}{\omega(\kappa + i\omega)} \right)] \quad (4.12)$$

where $A_x + iA_y = A_1$. Then by using the Euler's formula $e^{it} = \cos t + i\sin t$, (4.12) could be rewritten as

$$\begin{aligned} & \frac{A_x}{\omega(\kappa^2 + \omega^2)(T - t)} \{ -\kappa\omega \cos(\omega t) e^{-\kappa(T-t)} - \kappa^2(\sin(\omega T) - \sin(\omega t)) \\ & \quad - \omega^2 \sin(\omega t) (e^{-\kappa(T-t)} - 1) + \kappa\omega \cos(\omega T) + \\ & \frac{A_y}{\omega(\kappa^2 + \omega^2)(T - t)} \{ \kappa\omega \sin(\omega t) e^{-\kappa(T-t)} - \kappa^2(\cos(\omega T) - \cos(\omega t)) \\ & \quad - \omega^2 \cos(\omega t) (e^{-\kappa(T-t)} - 1) - \kappa\omega \sin(\omega T) \\ & = A_x z_{3j,t} + A_y z_{4j,t} \end{aligned} \quad (4.13)$$

with

$$z_{3j,t} = \frac{1}{\omega(\kappa^2 + \omega^2)(T-t)} \{-\kappa\omega \cos(\omega t) e^{-\kappa(T-t)} - \kappa^2(\sin(\omega T) - \sin(\omega t)) - \omega^2 \sin(\omega t) (e^{-\kappa(T-t)} - 1) + \kappa\omega \cos(\omega T)\} \quad (4.14)$$

$$z_{4j,t} = \frac{1}{\omega(\kappa^2 + \omega^2)(T-t)} \{\kappa\omega \sin(\omega t) e^{-\kappa(T-t)} - \kappa^2(\cos(\omega T) - \cos(\omega t)) - \omega^2 \cos(\omega t) (e^{-\kappa(T-t)} - 1) - \kappa\omega \sin(\omega T)\} \quad (4.15)$$

III.2.2. The Two Terms Fourier Model

Based on the above Fourier model from Moreno, Novales and Platania (2013), we extend the one term Fourier model to a two terms Fourier model by keeping two terms of the Fourier series in the reversion mean. The first equation in (4.11) could be extended as

$$Y_{j,t} = \delta_1 z_{1j,t} + \delta_2 z_{2j,t} + \delta_3 z_{3j,t} + \delta_4 z_{4j,t} + \delta_5 z_{5j,t} + \delta_6 z_{6j,t} + u_{j,t} \quad (4.16)$$

The expressions of parameters $z_{3j,t}$ and $z_{4j,t}$ are the same as in equations (4.14) and (4.15) with the change of notation ω to ω_1 . The parameters according to the second term of Fourier series could be formulated as

$$z_{5j,t} = \frac{1}{2\omega_2(\kappa^2 + 4\omega_2^2)(T-t)} \{-2\kappa\omega_2 \cos(2\omega_2 t) e^{-\kappa(T-t)} - \kappa^2(\sin(2\omega_2 T) - \sin(2\omega_2 t)) - 4\omega_2^2 \sin(2\omega_2 t) (e^{-\kappa(T-t)} - 1) + 2\kappa\omega_2 \cos(2\omega_2 T)\} \quad (4.17)$$

$$z_{6j,t} = \frac{1}{2\omega_2(\kappa^2 + 4\omega_2^2)(T-t)} \{2\kappa\omega_2 \sin(2\omega_2 t) e^{-\kappa(T-t)} - \kappa^2(\cos(2\omega_2 T) - \cos(2\omega_2 t)) - 4\omega_2^2 \cos(2\omega_2 t) (e^{-\kappa(T-t)} - 1) - 2\kappa\omega_2 \sin(2\omega_2 T)\} \quad (4.18)$$

The two term expansion model should have more flexibility on capturing the dynamics of the yield curve than the one term model.

III.2.3. Estimation Method

In this research, the empirical analysis of the two terms Fourier model is not given, since there will be a problem with the degrees of freedom in the estimation of the two terms Fourier. The cross-sectional data sample in this research is not long enough to estimate the large amounts of parameters in the two terms expansion model. Therefore, the empirical analysis is reported with only results from the one term Fourier model.

We have written the in-sample fitting model in form of a regression model as given in equation (4.11). However, this model could not be estimated as a simple regression model since the explanatory variables are dependent on the structural parameters κ and ω . So the regular regression method cannot solve the problem with all the observation. As to this nonlinear optimization problem, Moreno, Novales & Platania (2013) estimate the model day by day. They use the everyday cross-sectional data of the interest rates to estimate the parameters $\kappa, \omega, \delta_1, \delta_2, \delta_3, \delta_4$ for each day, within the values of those, the minimum values of the sum of squared residuals in (4.11) are achieved. The sum of squared residuals in equation (4.11) is given as below:

$$SR(\hat{\theta}_t) = \sum_{j,t} [Y_{j,t} - (\delta_1 z_{1j,t} + \delta_2 z_{2j,t} + \delta_3 z_{3j,t} + \delta_4 z_{4j,t})]^2 \quad (4.19)$$

After the day-by day estimation for each day, the time series of $\alpha, \sigma^2, A_x, A_y, \kappa$ and ω could be generated respectively and we express the five time series as a structural parameter denoted by $\theta = (\alpha, \sigma^2, A_x, A_y, \kappa, \omega)$. Since the Vasicek model is nested in the Fourier model by setting $z_{3j,t} = z_{4j,t} = 0$ in (4.11), the same estimation method is applied to the Vasicek model and the structural parameters $\theta = (\alpha, \sigma^2, \kappa)$ are estimated.

III.2.4. Prediction Approach

The interest rate prediction is conducted by building parameter forecasting using a first-order autoregression. The parameters in the Fourier model are assumed to follow a first-order autoregressive process as a vector denoted by $\hat{\theta}$,

$$\hat{\theta} = \hat{c} + \gamma \hat{\theta}_{t-1} + \varepsilon_t \quad (4.20)$$

where ε_t is white noise. As the Fourier is a short rate model based on the single factor of the instantaneous rate, we construct the prediction of it by using the Euler discretization,

$$E[r_{t+\Delta t}|r_t] = r_t + \kappa(\mu - r_t)\Delta t \quad (4.21)$$

In this equation, Δt denotes the required forecast horizons set at 1, 5, and 21 in responding to one-day, one-week and one-month ahead forecasting. Also μ is a nonlinear function of the structural parameters in the Fourier model while it is a constant parameter in the Vasicek model. With the prediction of parameters and the instantaneous rate, the interest rates with the other maturities $R(r_t, t, T)$, could be obtained by using equation (4.11).

III.3. Empirical Analysis

Chinese inter-bank Zero-coupon yields is used with 2335 daily observations for each maturity from March 1st, 2006 to June 30th, 2015. Yields maturities included are 1, 3, 6 months and 1, 2, 3, 5, 7, 10, 20 and 30 years. The data source is ChinaBond and the yield curve is constructed by using bootstrapping on the coupon bonds in inter-bank market and Hermite interpolation is applied to smooth the yields as stated in ChinaBond. And the yields are annualized continuously compounded and given in percentage.

The Fourier model is estimated by nonlinear optimisation with cross-sectional data and the Vasicek is estimated in the same way as a special case of the Fourier extension model. A good model of yield curve should provide both good in-sample fitting and satisfactory out-of-sample prediction. Therefore, we estimate the same models for out-of-sample forecasting as well.

III.3.1. Data Description

The summary statistics of Chinese zero-coupon yields are reported in Table III-1. The mean value increases gradually as the maturity moves longer. The yields mean is around

2.3 with maturity of one month while the value of mean is about 4.2 at 30-year maturity. This result indicates that the average Chinese treasury yield curve is upward sloping as expected. As given in the third column, the value of standard deviation has a decreasing trend with maturity which illustrates that the short-term yields are much more volatile than the long-term yields. The standard deviation of 30-year is 0.0079 which is less than half of the standard deviation of yields of 1-month. This result is consistent with most findings based on the U.S. market, since both slope and curvature add volatility to the short end of yield curve while volatility of level is the only factor which influences the long end volatility. According to the distribution statistics, the results show a lack of symmetry and a flatter distribution than Gaussian in the data. The value of skewness indicates the asymmetry from normal distribution, with positively skewed yields at maturities of one month and longer than 5 years, and negatively skewed yields at maturities from 3-month to 3-year. The last two columns display the autocorrelations of the yields with different maturities at displacements of 5 and 30 days respectively. The Chinese yields prove high level of persistence at all maturities. The autocorrelations are all above 0.8 at both displacements. The 5-th order autocorrelations of the sample are all over 0.98 while the 30-th range from 0.807 to 0.906. And the medium-term yields show slightly higher persistence than the short and long end of the yield curve at both displacements. The high persistence of yields at all the maturity and relatively higher persistence at medium maturity is also consistent with the findings of research based on the U.S. market.

Table III-1: Summary statistics of Chinese zero-coupon yields

Month	Mean	Std. Dev.	Kurtosis	Skewness	Min.	Max.	$\hat{\rho}(5)$	$\hat{\rho}(30)$
1	2.3140	0.0194	0.6263	0.6105	0.7102	6.5750	0.988	0.807
3	2.4896	0.0179	-0.6754	-0.1324	0.7989	5.1132	0.997	0.874
6	2.5479	0.0172	-0.8496	-0.2647	0.8183	4.3744	0.998	0.886
12	2.6440	0.0168	-0.8635	-0.3232	0.8871	4.2503	0.998	0.892
24	2.8554	0.0160	-0.7770	-0.2917	1.0700	4.4190	0.999	0.906
36	3.0281	0.0142	-0.6813	-0.2024	1.2437	4.5003	0.998	0.891
60	3.2887	0.0120	-0.8364	0.0180	1.7342	4.5293	0.998	0.870

84	3.4962	0.0109	-0.7953	0.0580	2.1223	4.6698	0.998	0.863
120	3.6753	0.0096	-0.8642	0.2937	2.6711	4.7222	0.997	0.855
240	4.1204	0.0080	-0.2348	0.4091	3.3782	5.0968	0.998	0.831
360	4.2349	0.0079	-0.3064	0.3665	3.4800	5.1988	0.997	0.834

III.3.2. In-sample Fitting of Yield Curve

The in-sample fitting results are given in this section. Since the data sample from 03/01/2006 to 06/30/2015 covers the time period of the financial crisis and global recession happened in 2008. In order to explore and understand the level effects of the financial crisis in 2008 on the Chinese government bond market, the yield curve will be fitted with a whole sample period from 03/01/2006 to 06/30/2015 and also a post-crisis period from 04/01/2009 to 06/30/2015 respectively. A better way to compare financial crisis influence on the interest rates is to fit the model within one whole period and one pre-crisis period due to the avoidance of choosing the cut-off date of post-crisis. However, the pre-crisis sample period is relatively short with only two-year data and the unreliable data set may provide unstable estimates. Thus, we use the post-crisis period for comparison. The reason for cut-off the data sample at 04/01/2009 is that the Chinese economy is widely considered to recovery gradually from April 2009 due to the 4 trillion-yuan stimulus program by the government.

Table III-2 displays the parameters estimation from the in-sample fitting for both whole period and post-crisis period. The parameter mean and standard deviation are given for both the Fourier model and the Vasicek model which is employed as benchmark. The minimized numerical value of the objective function and the sum of the absolute value of the pricing errors across all maturities are given in the last two rows in purpose of measuring the fitting ability of each model to the observed data. In both sample periods, the Fourier models show better in-sample fitting performances than the Vasicek models with significantly lower values of both minimized objective function and the sum of absolute pricing errors. The Fourier model cut down 55.5% of the sum of absolute pricing errors both in the whole sample period and the post-crisis period. Furthermore,

the sum of absolute pricing errors is reduced by 23.3% in the post-crisis period for both Fourier and Vasicek models than in the whole period. The better in-sample performance of both models over post-crisis period indicates significant impact of the global financial crisis to the Chinese yield curve.

Table III-2: Parameter mean from in-sample Fitting

Parameter	2006 - 2015		2009 - 2015	
	Fourier	Vasicek	Fourier	Vasicek
$\hat{\delta}_1$	0.0477 (0.0001)	0.0472 (0.0005)	0.0474 (0.0002)	0.0474 (0.0008)
$\hat{\delta}_2$	0.0004 (0.0000)	0.0014 (0.0000)	0.0002 (0.0000)	0.0007 (0.0001)
$\hat{\delta}_3$	0.0018 (0.0023)		0.0005 (0.0126)	
$\hat{\delta}_4$	-0.0079 (0.0022)		0.0026 (0.0111)	
$\hat{\kappa}$	0.2435 (0.0010)	0.3373 (0.0166)	0.2290 (0.0014)	0.2743 (0.0227)
$\hat{\omega}$	4.9044 (0.0232)		5.2498 (0.0310)	
$\sum_{i,t} \min SR$	0.0244	0.1051	0.0198	0.0861
$\sum_{i,t} \hat{u}_{i,t} $	13.8489	31.1426	10.6244	23.8960

Note: The parameter standard deviations are indicated in the parentheses.

The goodness of fit statistics of yields with each maturity is given in Table III-3 for both models within two sample periods respectively. According to the estimation result, both Fourier and Vasicek models fit the yield curve fairly well with low pricing errors at each maturity and the Fourier model outperforms the Vasicek with lower numerical pricing errors at all maturities. For example, the sum of squared errors lies below 0.006 in Fourier model while in the Vasicek model all the sum of squared errors at different maturity are over 0.006 in full sample. Interestingly, both models provide much better approximation of the data within the post-crisis sample period than in the whole period.

The pricing errors over all the maturities are relatively lower by using the post-crisis period than the whole period for both Fourier and Vasicek models except for the one-year error in Fourier. Thus, the goodness of fit result confirms the conclusion that the Chinese yield curve is very sensitive to the global financial crisis.

Table III-3: Goodness of Fit of Yields with Each Maturity

Maturity	2006 - 2015				2009 - 2015			
	Fourier		Vasicek		Fourier		Vasicek	
	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $
3M	0.0055	1.9684	0.0202	4.6675	0.0038	1.4620	0.0142	3.3448
6M	0.0031	1.3633	0.0215	4.8014	0.0021	1.0246	0.0171	3.6527
1Y	0.0031	1.3886	0.0182	4.1063	0.0059	1.1421	0.0163	3.4461
2Y	0.0031	1.5893	0.0117	3.2030	0.0023	1.1190	0.0108	2.7737
3Y	0.0027	1.4361	0.0091	2.7131	0.0023	1.1240	0.0079	2.0772
5Y	0.0016	1.2191	0.0068	2.7412	0.0016	0.9839	0.0058	2.0598
7Y	0.0010	1.0993	0.0043	2.1752	0.0008	0.7567	0.0035	1.5283
10Y	0.0015	1.4564	0.0035	2.0436	0.0013	1.1724	0.0031	1.5927
20Y	0.0012	1.1681	0.0035	2.0569	0.0010	0.9247	0.0027	1.4959
30Y	0.0017	1.1599	0.0062	2.6345	0.0016	0.9150	0.0047	1.9248

The evolution of estimated parameters from both Vasicek and Fourier models within the whole sample period from 03/01/2006 to 06/30/2015 are plotted in Figure III-1 and Figure III-2. And the behaviour of the parameters from the post-crisis sample period from 04/01/2009 to 06/30/2015 are given in Figure III-3 and Figure III-4 respectively.

In the whole sample period, the parameters estimated from the Vasicek model are relatively more volatile than those estimated from the Fourier model. The σ^2 estimated from the Vasicek model varies between 0 and 0.01 with an imported upper boundary of 0.01 while the σ^2 estimated from the Fourier model is below 0.005 with no externally setted boundary. Also, the speed of reversion denoted by κ moves

between 0 and 1 in the Vasicek model with an upper edge set at 1. In contrast, it lies in the range from 0.05 to 0.55 in the Fourier model. The behaviour of α are similar in both models.

Figure III-1: Estimated Parameters from Vasicek Model within the Whole Sample

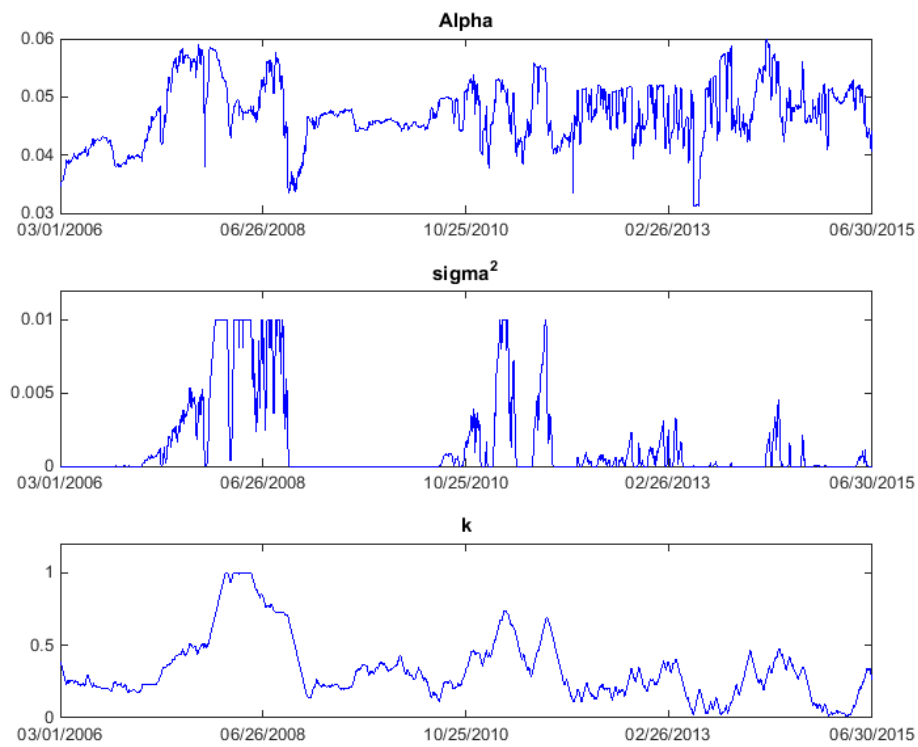
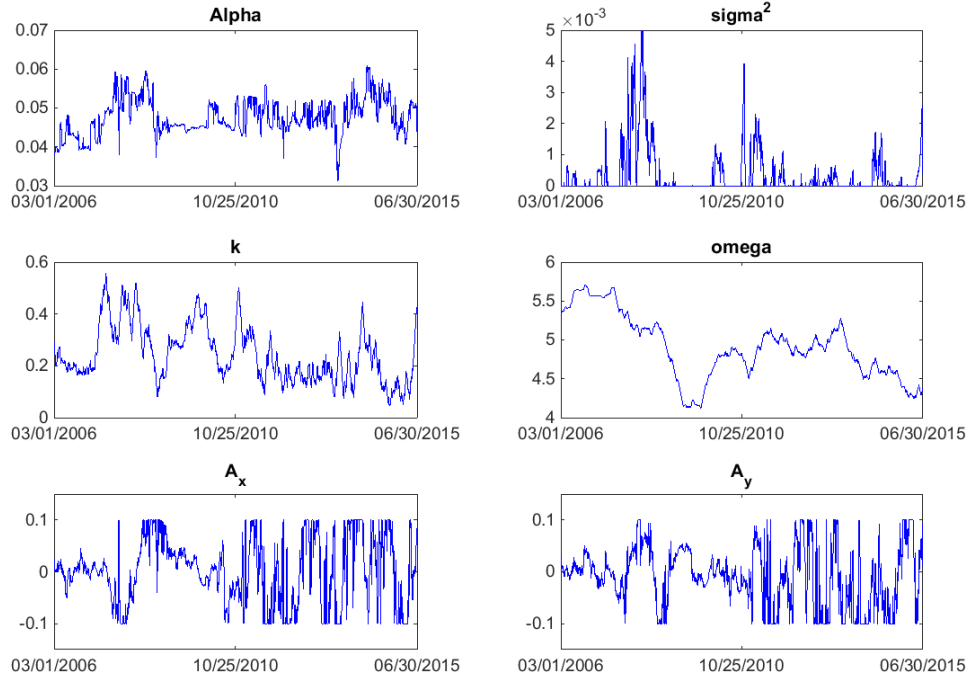


Figure III-2: Estimated Parameters from Fourier within the Whole Sample



In the post-crisis sample period, the Fourier model also provides a relatively more stable estimate of the parameters than the Vasicek model especially for the σ^2 and κ . In contrast to the results from the whole sample estimation, the time evolution of estimated parameters from the Vasicek model are very close while the estimates from the Fourier model show observable differences in the dynamics of ω , A_x and A_y . For example, the ω from the whole sample period estimation lies within the range between 4.1 and 5.2 from 04/01/2009 to 06/30/2015 and the estimates from the post-crisis sample period moves around mean value of 5.25 with fluctuations up to 0.3. The estimates of A_x and A_y in both sample periods are all bounded between -0.1 and 0.1 for model stability and distinct movements within the same time period are observed. All these differences on the estimated behaviours of the additional parameters in the Fourier model indicate that the 2008 global financial crisis has a strong effect on the Chinese term structure of interest rates and its dynamics. Also, we can conclude that the Fourier model could capture the in-depth influence from the global economy to the Chinese yield curve.

Figure III-3: Estimated Parameters from Vasicek within Post-crisis Period

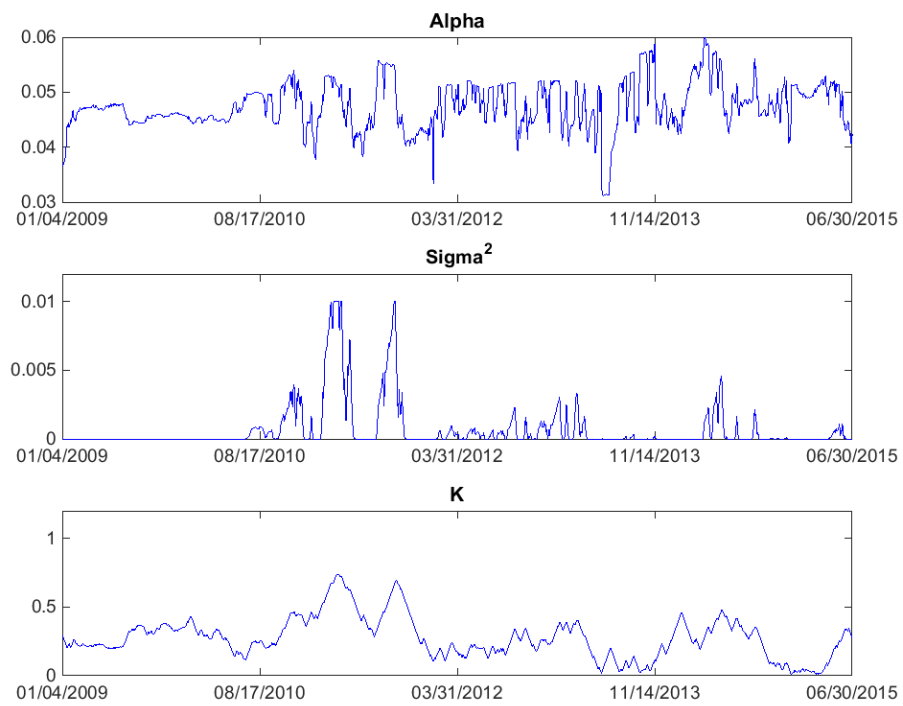
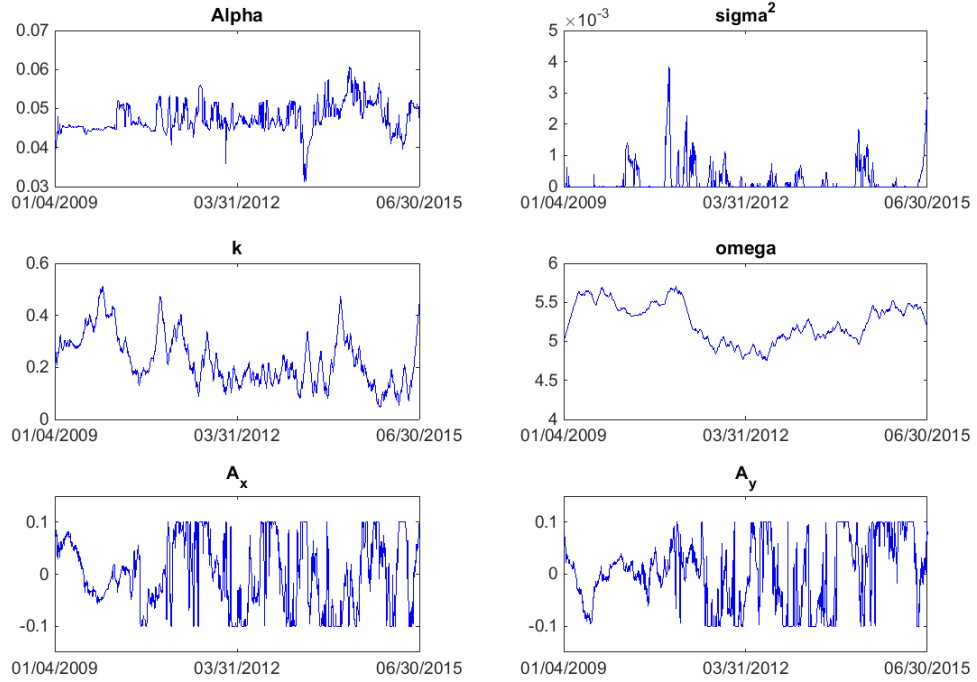


Figure III-4: Estimated Parameters from Fourier within Post-crisis Period



III.3.3. Out-of-Sample Forecasting

A satisfactory model should not only provide a good in-sample approximation to the yield curve fitting, but also offer satisfactory out-of-sample forecasts. In order to explore the forecasting power of the Fourier model under different situation, we choose three time periods with various shapes of yield curve as the forecasting periods which covers the slots from 12/11/2008 to 12/25/2009, 02/27/2012 to 03/12/2013 and 06/16/2014 to 06/30/2015. Each forecasting period contains 260 trading days and roughly covers one year. Over a given forecasting period, both Vasicek and Fourier models are estimated from 03/01/2006 to each day and we take every estimated parameter time series up to that day to estimate vector autoregression with order one and forecast the value of each parameter within the forecasting horizons of one-day, one-week and one-month respectively.

During the first forecasting period from Dec. 2008 and Dec. 2009, the Chinese economy touched the bottom and began to show signs of recovery from the global financial crisis.

As displayed in

Figure III-5, the yield curve is steeply upward sloping with very low rates at the short-end. Also, approximately from Sep. 2009, the level of yield curve at all the maturities moves up slightly which might be an indication of full recovery. We are interested to see if the model provides a satisfactory forecast for the Chinese yield curve even when the financial crisis occurs.

Figure III-5: 3-D Plot of Yield Curve of the First Forecasting Period (12/11/2008-12/25/2009)

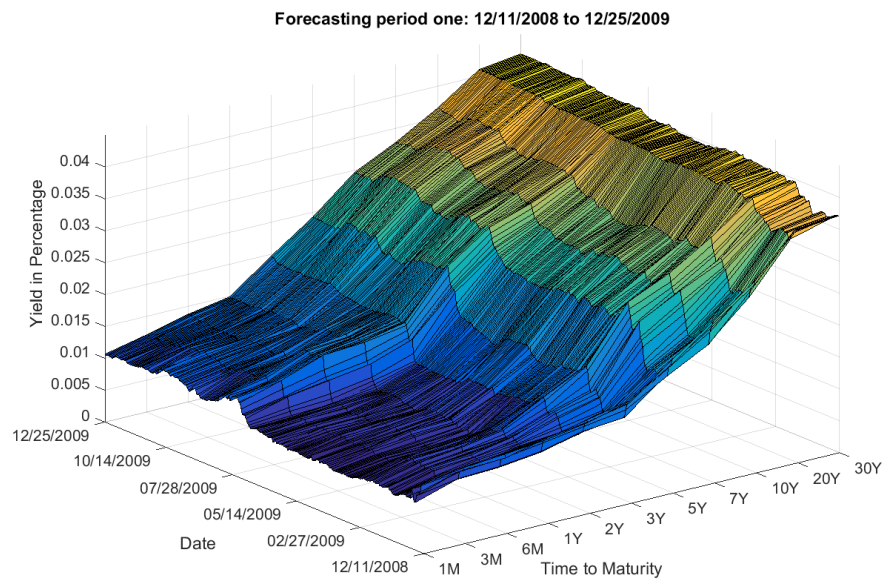


Table III-4, Table III-5 and Table III-6 give the 1-, 5- and 21-day ahead forecasting errors for the first period respectively. In the 1-day ahead forecasts, the Fourier model outperforms the Vasicek at all maturities except for the one-month yield. Also the Fourier model only produces higher values of sum of squared forecasting errors and sum of absolute forecasting errors than the Vasicek model at one-month and three-month yields in 5-day ahead forecasts, and at one-month, three-month and six-month yields in 21-day ahead forecasts. These results illustrate that the Fourier model process good and better out-of-sample forecasting than Vasicek model during the time of global financial crisis.

Table III-4: One-day Ahead Forecasting Errors in Period 1

Maturity	Fourier		Vasicek	
	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $
1M	0.0215	49	0.0209	46
3M	0.0436	83	0.0730	107
6M	0.0559	84	0.1639	175
1Y	0.1597	172	0.5901	342
2Y	0.8378	438	1.0375	559
3Y	0.2635	196	0.4179	257
5Y	0.1894	190	0.4192	301
7Y	0.1467	159	0.3655	266
10Y	0.1050	113	0.1401	139
20Y	0.1100	137	0.1662	159
30Y	0.1258	163	0.2614	240
$\sum_{i,t} \hat{u}_{i,t}^2$	2.0588		3.9256	
$\sum_{i,t} \hat{u}_{i,t} $		1784		2592

Table III-5: One-week Ahead Forecasting Errors in Period 1

Maturity	Fourier		Vasicek	
	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $
1M	0.1406	131	0.1219	116
3M	0.1628	146	0.1233	133
6M	0.2390	145	0.2694	213
1Y	0.2800	179	0.7236	385
2Y	0.9848	472	1.4666	581
3Y	0.4806	247	0.6494	296
5Y	0.4490	235	0.7232	346
7Y	0.3842	228	0.6373	324
10Y	0.2678	169	0.3321	200

20Y	0.1563	154	0.2381	182
30Y	0.1521	170	0.2805	243
$\sum_{i,t} \hat{u}_{i,t}^2$	3.6974		5.5655	
$\sum_{i,t} \hat{u}_{i,t} $		2276		3019

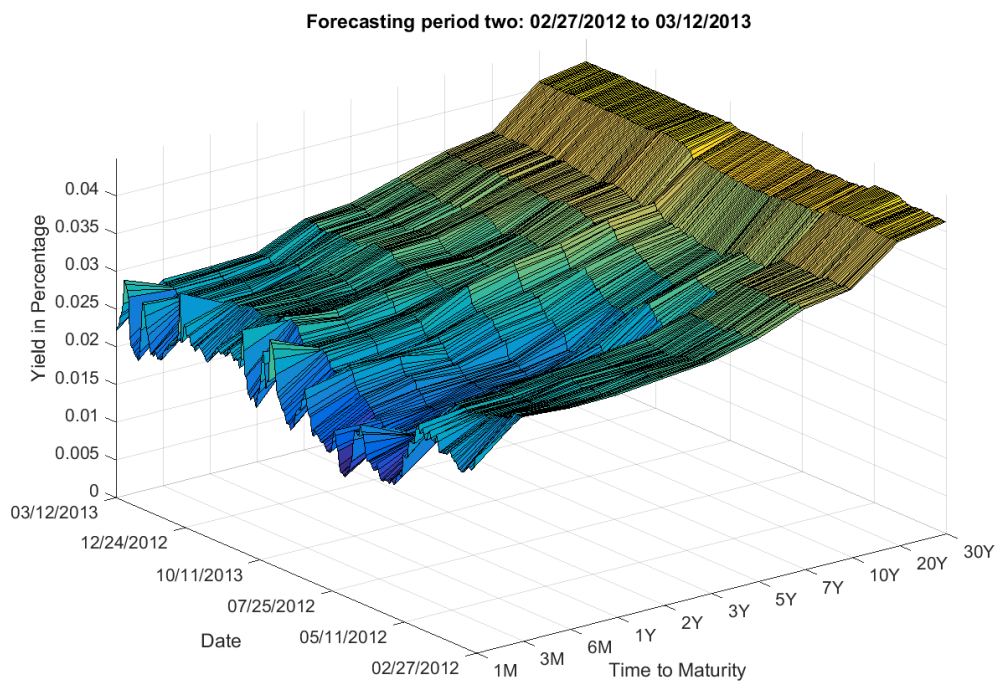
Table III-6: One-month Ahead Forecasting Errors in Period 1

Maturity	Fourier		Vasicek	
	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $
1M	2.0719	524	1.5308	400
3M	2.2699	521	1.3037	358
6M	2.8792	511	1.7816	497
1Y	2.2931	479	2.4763	661
2Y	3.1212	750	3.5511	793
3Y	2.7439	607	3.2641	625
5Y	2.1986	530	3.1476	631
7Y	1.7127	484	2.5372	597
10Y	1.0775	387	1.4265	436
20Y	0.4235	213	0.5668	251
30Y	0.2511	206	0.3932	272
$\sum_{i,t} \hat{u}_{i,t}^2$	21.0426		21.9791	
$\sum_{i,t} \hat{u}_{i,t} $		5210		5547

As shown in Figure III-6, the Chinese yield curve in the second forecasting period from 02/27/2012 to 03/12/2013 is relatively flat and stable at the medium and long terms while it is much more volatile at the short end. Significant fluctuations within 140 basis points can be observed at one-month interest rate over time. From February to April 2012, the whole yield curve moves up with term spread less than 35 basis points due to the relatively high CPI in the beginning of year 2012. From May to July, the treasury yields with all the maturities decrease slightly especially at the short end and the yield

curve moves down steeply. This is mostly caused by the monetary policy. The central bank cut the reserve ratio by 0.5% twice and also moved down the official one-year saving and borrowing rate. In the following months to the end of 2012, the economy appears steady rise and drives the treasury yields pick up with fluctuations. In the beginning of 2013, the yield curve stays stable with flat shape. We choose this time period to explore if the Fourier model can provide well prediction even when the information on the instantaneous rate is not reflected on the medium and long term yields.

Figure III-6: 3-D Plot of Yield Curve of the Second Forecasting Period (12/11/2008-12/25/2009)



The forecasting errors for both models within three different horizons from 02/27/2012 to 03/12/2013 are shown in Table III-7, Table III-8 and Table III-9. The Fourier again delivers more accurate prediction to the observed data than the Vasicek by largely reducing the predicting errors. At the one-day forecasting horizon, the aggregate sum of squared errors is cut down by 78.2% when incorporating the Fourier series to the Vasicek model and the aggregate sum of absolute errors is reduced by 54.8%. These errors are brought down by 60% and 43.1% at one-week horizon and, 22.6% and 21%

at one-month forecast horizon. The superiority of the Fourier model weakens as the predicting horizon lengthen. For the forecasts of each yield through all maturities, the Fourier model wins over all the three horizons, only with two exceptions. At the one-day forecasting horizon, the Fourier and Vasicek seem to provide a close performance on predicting the one-month yield. The other particular case is the one-month yield prediction at one-month forecasting horizon, at which the Vasicek shows a slightly better prediction.

Table III-7: One-day ahead forecasting errors in period 2

Maturity	Fourier		Vasicek	
	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $
1M	0.3798	190	0.3826	189
3M	0.4418	268	2.6030	746
6M	0.1851	185	1.7747	616
1Y	0.0870	119	1.0738	476
2Y	0.1283	152	0.4839	302
3Y	0.1069	133	0.1846	168
5Y	0.0718	108	0.3244	249
7Y	0.0514	95	0.1500	167
10Y	0.1622	182	0.5295	336
20Y	0.0874	135	0.1958	181
30Y	0.0354	76	0.2833	205
$\sum_{i,t} \hat{u}_{i,t}^2$	1.7371		7.9855	
$\sum_{i,t} \hat{u}_{i,t} $		1643		3635

Table III-8: One-week ahead forecasting errors in period 2

Maturity	Fourier		Vasicek	
	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $
1M	2.0120	529	2.0789	535

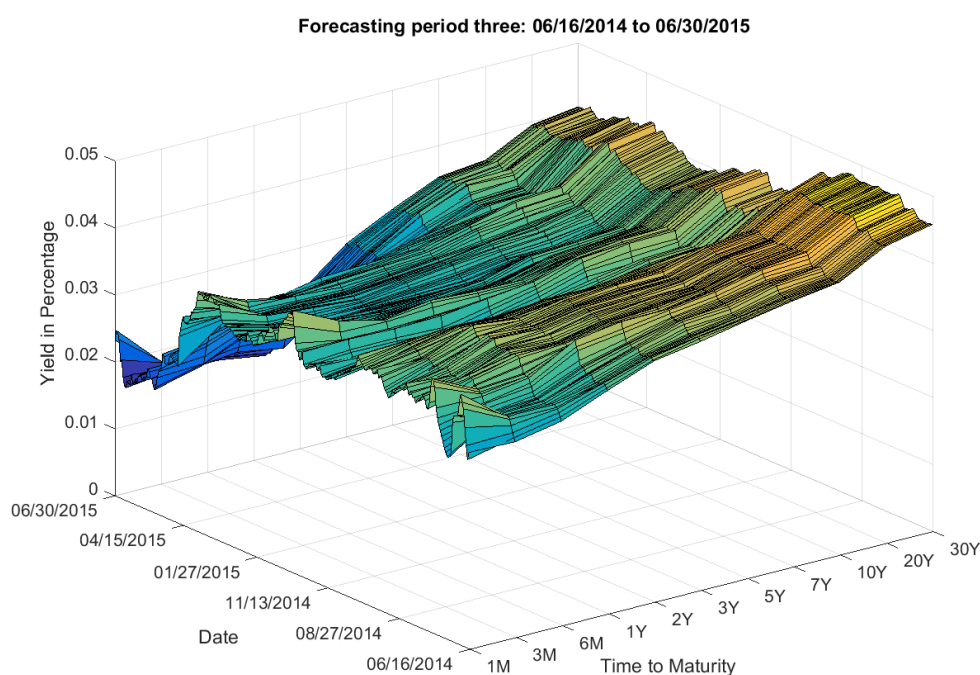
3M	0.4410	244	2.5427	716
6M	0.4420	273	1.7690	591
1Y	0.2273	182	1.0822	470
2Y	0.2719	199	0.5685	314
3Y	0.2586	190	0.3302	226
5Y	0.2200	172	0.4902	285
7Y	0.1000	130	0.2213	194
10Y	0.1894	185	0.5952	346
20Y	0.1106	147	0.2745	215
30Y	0.0509	92	0.3316	226
$\sum_{i,t} \hat{u}_{i,t}^2$	4.3237		10.2842	
$\sum_{i,t} \hat{u}_{i,t} $		2342		4118

Table III-9: One-month ahead forecasting errors in period 2

Maturity	Fourier		Vasicek	
	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $
1M	3.5760	775	3.3373	736
3M	1.3559	449	2.7381	725
6M	1.9111	547	2.1721	635
1Y	1.0929	381	1.6127	533
2Y	1.1212	396	1.2869	443
3Y	1.1416	391	1.2440	435
5Y	0.7010	316	0.9821	416
7Y	0.3158	214	0.4509	278
10Y	0.3038	227	0.7133	371
20Y	0.1549	169	0.2982	224
30Y	0.0869	116	0.3500	243
$\sum_{i,t} \hat{u}_{i,t}^2$	11.7610		15.1855	
$\sum_{i,t} \hat{u}_{i,t} $		3981		5039

The third forecasting period covers one year from 06/16/2014 to 06/30/2015. As shown in Figure III-7, the Chinese government yield curve displays a gradually decreasing trend with stable slope from June 2014 to February 2015 and a significant decline occurred from March 2015 at the short and medium term. To keep the financing cost down and promote the development of real economy sustainable, the central bank cut down the reserve ratio three times in February, April and June 2015 by 0.5%, 1% and 0.5% respectively. At the same time, the official one-year deposit rate and loan rate are moved down four times from 3% to 2% and 6% to 4.85% within 8 months. We are interested in investigating the prediction capacity of the Fourier model when a significant change is happened on the slope of the yield curve.

Figure III-7: 3-D Plot of Yield Curve of Forecasting Period 3 (06/16/2014-06/30/2015)



As displayed in Table III-10, Table III-11 and Table III-12, the forecasting errors in the third period has been largely reduced by introducing the Fourier series to the conventional Vasicek model. At one-day ahead forecasting horizon, the aggregate absolute value of pricing errors is cut down by 78.8% and the aggregate squared errors is reduced by 53.6%. In contrast, the pricing errors are brought down by 54.3% and

36.7% at one-week forecasting horizon and 19.4% and 12.4% at one-month horizon. These results indicate that the Fourier model outperforms the Vasicek model, especially providing better predictions at short-horizon.

Table III-10: One-day ahead forecasting errors in period 3

Maturity	Fourier		Vasicek	
	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $
1M	0.2231	159	0.2245	159
3M	0.2459	190	1.0543	391
6M	0.1187	125	1.7417	558
1Y	0.2336	181	1.9043	549
2Y	0.1085	136	1.0436	413
3Y	0.2222	193	0.7633	332
5Y	0.1319	141	0.4582	279
7Y	0.1388	142	0.3595	251
10Y	0.2331	212	0.4525	298
20Y	0.0896	123	0.3694	214
30Y	0.1057	126	0.4546	282
$\sum_{i,t} \hat{u}_{i,t}^2$	1.8512		8.7260	
$\sum_{i,t} \hat{u}_{i,t} $		1728		3725

Table III-11: One-week ahead forecasting errors in period 3

Maturity	Fourier		Vasicek	
	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $
1M	1.6041	450	1.6224	453
3M	0.5710	299	1.1473	424
6M	0.3630	241	1.6578	549
1Y	0.6116	281	1.9365	578
2Y	0.2824	220	1.1078	438

3Y	0.3224	245	0.8504	367
5Y	0.2576	199	0.5891	329
7Y	0.2559	198	0.4837	292
10Y	0.4050	264	0.6806	355
20Y	0.2368	188	0.5177	278
30Y	0.2846	213	0.7646	359
$\sum_{i,t} \hat{u}_{i,t}^2$	5.1945		11.3577	
$\sum_{i,t} \hat{u}_{i,t} $		2798		4422

Table III-12: One-month ahead forecasting errors in period 3

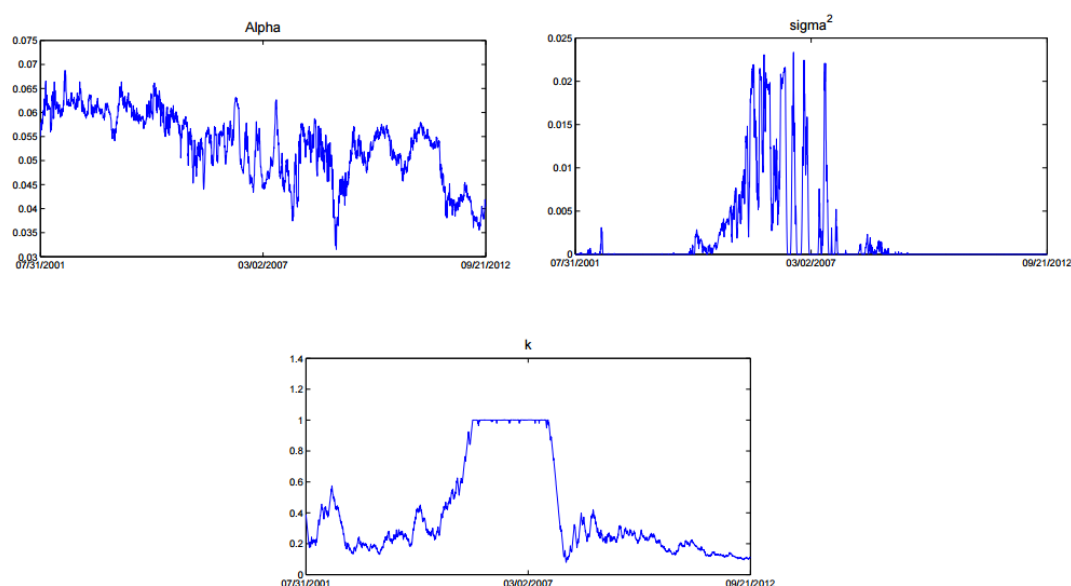
Maturity	Fourier		Vasicek	
	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $	$\sum_t \hat{u}_t^2$	$\sum_t \hat{u}_t $
1M	5.0548	897	5.0568	880
3M	3.2089	708	3.5893	734
6M	2.9467	660	3.3855	769
1Y	3.2870	671	3.9620	819
2Y	1.7477	557	2.4712	651
3Y	1.1282	446	1.8582	538
5Y	0.9791	411	1.5634	506
7Y	0.9544	428	1.4299	496
10Y	1.3274	466	1.8206	560
20Y	0.8319	373	1.2540	438
30Y	0.9650	408	1.4775	488
$\sum_{i,t} \hat{u}_{i,t}^2$	22.4311		27.8388	
$\sum_{i,t} \hat{u}_{i,t} $		6024		6879

III.3.4. Comparison with the Study in U.S.

In this section, we compare our results with findings from the U.S. who uses the same method, to point out both similarities and differences on the dynamics of yield curve.

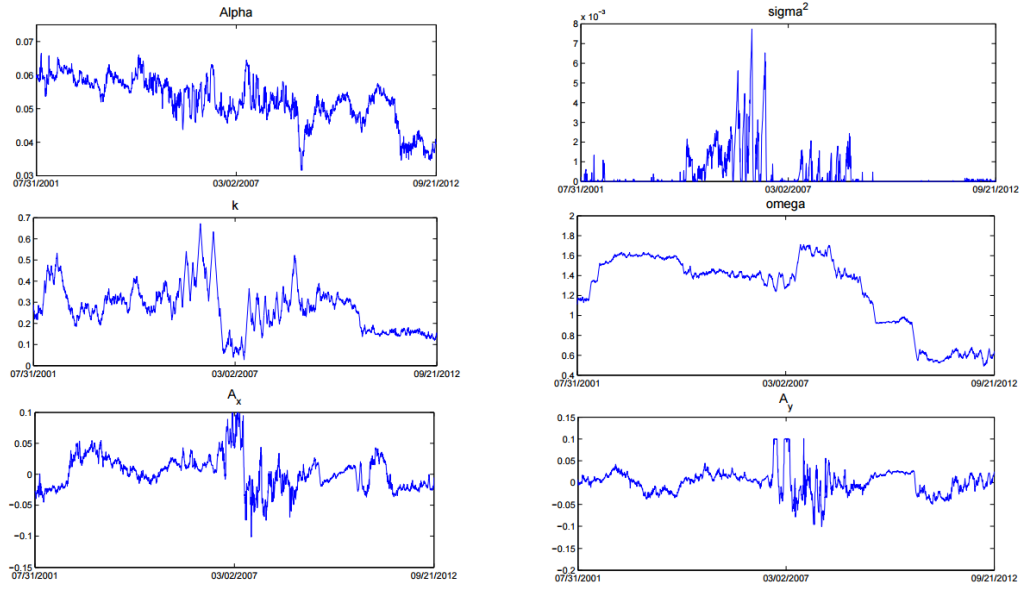
Moreno, Novales and Platania (2013) find similar in-sample fitting results with us. The Fourier model performs better in-sample fitting to the U.S. data than Vasicek. The aggregate sum of squares is cut down by 76% relative to the Vasicek in the U.S. while that is found to be 55.5% in China. Figure III-8 and Figure III-9 display the estimated parameters from both Vasicek and Fourier models in the U.S.. As to the results in Vasicek model, we find that the long-run equilibrium mean α varies between 0.03 and 0.07 in both studies. In addition, high and volatile volatility (σ^2) can be observed around 2007-2008. This indicates the financial crisis impact on the yield curve in both countries. Furthermore, during the 2008 financial crisis, the mean-reversion speed k is high as well. But the reversion speed in China seems a little bit lower than that of the U.S. It may illustrate that the financial crisis shock to the yield curve is more significant in the U.S. than in China. According to the estimation results of the Fourier model, the volatility is largely reduced in both countries. The assumption of Fourier mean can help capture the volatility on both U.S. and Chinese yield curves. However, the Fourier coefficients show different evolutions. They are much more volatile in China than those in the U.S. As to the out-of-sample forecasting, the Fourier model outperforms the Vasicek in both studies.

Figure III-8: Estimated Parameters from Vasicek in U.S.



Source: Moreno, Novales and Platania (2013)

Figure III-9: Estimated Parameters from Fourier in U.S



Source: Moreno, Novales and Platania (2013)

III.4. Summary and Conclusion

In this paper, we introduce the Fourier extension of the classic Vasicek model to describe the term structure of interest rates in China. In the Fourier model, the instantaneous rate expressed by a stochastic process is assumed to revert to the long-run mean which follows a Fourier series. The single-factor-based characteristic ensures the simplicity. The incorporated Fourier series is capable to describe the cyclical behaviour of the interest rate and allow more flexibility and tractability on capturing and estimating the movements of the yield curve. In the meantime, the Vasicek model is a special case of the Fourier model with different parameter setting. This fact brings us convenience on comparing the Fourier model with the Vasicek model to count the gain of the extension.

In addition, we dig deeper on the model construction by expanding the one term Fourier model to the two terms model. This expansion gives more flexibility on describing the movement of yield curve.

The empirical in-sample fitting and out-of-sample forecasting of both Vasicek model and the Fourier extension model with only one term in the Fourier series are produced. The in-sample fitting is estimated by applying the nonlinear optimization technique with cross-sectional data day by day. In order to investigate the issue that if the Chinese yield curve is affected by the 2008 global financial crisis, we estimate the models with two sample periods. One period covers the whole data period from 2006 to 2015 and the other starts from 2009 to 2015 which represents the time period after financial crisis. The estimation results show that both Vasicek and the Fourier extension provide significantly better fitting to the data by using the post-crisis sample than the whole sample. It can be concluded that the 2008 financial crisis has influenced the Chinese term structure of interest rates evidently. Furthermore, the Fourier model indicates apparently more accurate approximation than the Vasicek in the in-sample fitting. In order to explore the predicting power of the Fourier model under different circumstances, the out-of-sample forecasting is conducted to three time periods in which the yield curve shows distinct moving dynamics. The empirical results illustrate that both Vasicek and Fourier models can provide good prediction to the Chinese term structure of interest rate but the Fourier model shows better and more reliable forecasting than the Vasicek under various economic background. And this superiority is much more apparent at shorter forecasting horizons.

By comparing our study with Moreno, Novales and Platania (2013), we find that the Fourier model provides better in-sample fitting and out-of-sample forecasting to both U.S. and Chinese yield curve than the Vasicek model. In addition, evidences are found in both studies on the financial crisis impact to the yield curve. And the financial crisis shock to the yield curve is more significant in the U.S. than in China. Furthermore, it can be concluded that the Fourier mean do help capture the moving dynamics on both U.S. and Chinese yield curves.

Overall, this study tests for the cyclical behaviour of interest rates in China and finds that such trends exist, evidence that the ‘financial’ superstructure of this mixed economy has adopted distinct market characteristics. The future research could be to expand the application of this model to the derivative pricing and risk management of

government bonds.

Chapter IV. Interactions between Chinese Government Bond Yields and the Economy

IV.1. Introduction

In the last chapter, the term structure of interest rates in China has been modelled without any linkage to the economy. The Fourier model is not related to any macroeconomic variable. The possible interaction between Chinese government bond yields and the macroeconomy is not investigated. In this chapter, we are interested in these questions. Is there any linkage between the Chinese government bond yields and the macroeconomy? Is it a one-way effect or two-way interaction? Does the addition of macroeconomic variables provide better estimation performance than the financial term structure models? Thus, we employ the so-called macro-finance model to explore the possible relationship.

The financial term structure models are all built for nominal bond yields only. The state variables in these dynamic models are called “latent factors” which are used to explain the term structure movements, but have no explicit economic meanings in the real world. For example, these latent factors are labelled as level, slope and curvature in Litterman and Scheinkman (1991), level, slope and butterfly in Dai and Singleton (2000). These latent factors describe the effects on the yield curve and not compared with any macroeconomic variable. Pearson and Sun (1994) use the name “short rate” and “inflation” to call the unobservable latent factors in their model, but in fact the inflation data are not used in the estimation and the macroeconomic variable inflation is not related directly to the yields model.

From the economic point of view, the term structure of interest rates contains important macroeconomic information and it has a close relationship with inflation and real activity. In addition, the short rate is a monetary policy instrument controlled by the central banks to achieve economic growth and stabilization goals. Although the financial term structure of interest rates models describes the movements of yield curve well, they cannot provide economic insights on the mechanism of the influences

between the macroeconomic variables and the term structure of interest rates since the linkage is not investigated directly.

The economists construct the vector autoregressive (VAR) model of bond yields and macroeconomic variables directly and try to explore the interaction between them empirically. For example, Estrella and Hardouvelis (1991) and Estrella and Mishkin (1998) both explore the one-way influence from the bond yields to macroeconomic variables. In contrast, Evans and Marshall (1998) analyse the macroeconomic effects on the yield curve. These macroeconomic studies focus on the one-way shock transmission from bond yields to macroeconomic variables or the reversion by using the VAR model of bond yields at some maturities and selected macroeconomic variables. The drawback of the economic VAR models is that they do not consider the bidirectional interaction between the bond yields and macroeconomy. In addition, the VAR models only allow observable variables to be inserted. The latent factors which describe the yield curve movement well are unobservable and could not be incorporated. The last and the most apparent disadvantage of the unrestricted VAR approach is that there is no interest rates theory behind and the yields at selected maturities included in the VAR cannot reflect the dynamics of the entire yield curve. However, the economic VAR models do benefit from the facility of using impulse response function and variance decomposition which are useful techniques providing deeper understanding of the interaction between the yield curve movements and the macroeconomic shocks.

Thus, to combine both financial and economic reviews and take the advantages from both sides will help explore the relationship between yield curve movements and macroeconomy. The macro-finance models bring macroeconomic variables or macroeconomic structure directly to the financial term structure models to investigate the interaction by keeping the micro-variation mechanism of term structure and including the macroeconomic variables.

Ang and Piazzesi (2003) firstly proposed the joint dynamics of yields on bonds with macroeconomic variables in VAR. They explore how macro variables affect bond prices and the dynamics of the yield curve by adding inflation and economic growth factors

to the affine latent factor model of term structure. They find that macro factors explain up to 85% of the variation in the short and middle parts of the yield curve, but at the long end of the yield curve unobservable factors still account for most of the movements. In Ang and Piazzesi (2003), only the unidirectional macro-to-yield effects are analysed since the output and inflation are modelled as completely exogenous to the yield curve.

Diebold, Rudebusch & Aruoba (2006) allow for the bidirectional interactions between the yield curve and macro economy in their model. The three-factor dynamic Nelson-Siegel term structure model proposed by Diebold and Li (2002) is combined with VAR dynamics for macro economy by including three macro variables inflation, real activity and the monetary policy. They claim that there exist bidirectional effects between the macroeconomy and the yield curve by using US data and the macro-to-yields effect is much more significant than the reverse.

Compare with developed economies, Chinese government bond market was established relatively late and has insufficient historical data for a long time. Thus the studies on macro-finance theory and modelling based on Chinese market is rather limited. Shi, Sun and Deng (2008) follow the macro-finance model proposed by Ichiue (2004) on CHIBOR to explore the predictability of term spread on observable macro factors. Their result indicates that changes of Chinese output growth, the inflation rate and the short rate could be predicted by yield curve and the macro variables also influent the yield curve. However, the yields data in their model might be unreliable. Wei (2008) applies the Rudebusch and Wu (2004) framework to the Chinese inter-bank government bond yields at maturities form one year to 20 years ranging from 2005 to 2008 and find that the level and slope factors could be explained by inflation and output respectively. Both Shi, Sun and Deng (2008) and Wei (2008)'s research are unable to test the yield curve effect on the macroeconomy. Wu, Jin and Zhang (2010) follows the model from Diebold, Rudebusch & Aruoba (2006) to investigate the possible interaction between yields curve and macroeconomy. They use monthly data of inter-bank government bond yields at maturity from one year to 5 years. The sample period is from January 2005 to September 2008. Three macroeconomic variables included are the growth of GDP, overnight inter-bank lending rate and CPI. They find that the level factor contains the

information of inflation and the slope factor could reflect the change of monetary policy. The curvature factor has insignificant economic meaning. Both Shi, Sun and Deng (2008) and Wu, Jin and Zhang (2010) employ the linear interpolation method to convert the quarterly GDP to monthly data and this approximation will largely increase the estimation error. Also, in Wu, Jin and Zhang (2010), less than four years' sample period with only five yields time series at various maturities might be not long enough for the macro-finance model with a large number of parameters to be estimated. More reasonable and constructive research is needed to explore the relationship between the yield curve and macro economy in China.

In this research, we construct the Nelson-Siegel form macro-finance model of Chinese market following Diebold, Rudebusch & Aruoba (2006). On one hand, this macro-finance model is an extension of dynamic Nelson-Siegel model which is flexible enough to capture various variation on the shape of yields curve and simple to be estimated. On the other hand, this Nelson-Siegel form macro-finance model benefits from exploring potential bidirectional effects between macroeconomy and yield curves. This work is very similar to Wu, Jin and Zhang (2010). But the sample period employed in this research is much longer than that in Wu, Jin and Zhang (2010). Instead of interpolating monthly GDP from quarterly data, the growth rate of industrial production is used to measure the real activity of the economy in this research.

The rest of this chapter is organized as below. Section 2 describe both yields-only and macro-finance models in state-space form. The estimation method is introduced in section 3. The two-step OLS and one-step maximum likelihood estimation approaches are presented. In section 4, the empirical results are given. The last section is the summary and conclusions.

IV.2. Model and Estimation Method

In this section, we present brief introductions of Nelson and Siegel (1987), Diebold and Li (2006) and Diebold, Rudebusch & Aruoba (2006). As to the estimation technique, the two-step OLS regression and one-step maximum likelihood via Kalman filter

methods are compared.

IV.2.1. Model Construction

The static Nelson-Siegel model

The Nelson and Siegel (1987) is a powerful and tractable yield curve model and very popular in practise both for investors and policy makers. The model provides simple and parsimonious estimation of a small number of parameters to describe the variation of the yield curve shapes. The Nelson-Siegel forward rate curve is constructed as the sum of a constant and a Laguerre function as below

$$f(\tau) = \beta_1 + (\beta_2 + \beta_3 \lambda) \exp(-\lambda \tau) \quad (5.1)$$

where $f(\tau)$ is the instantaneous forward rate in τ periods. The zero-coupon yield is an equally-weighted average of the instantaneous forward rates and the relationship could be expressed in form of

$$y(\tau) = \frac{1}{\tau} \int_0^\tau f(u) du \quad (5.2)$$

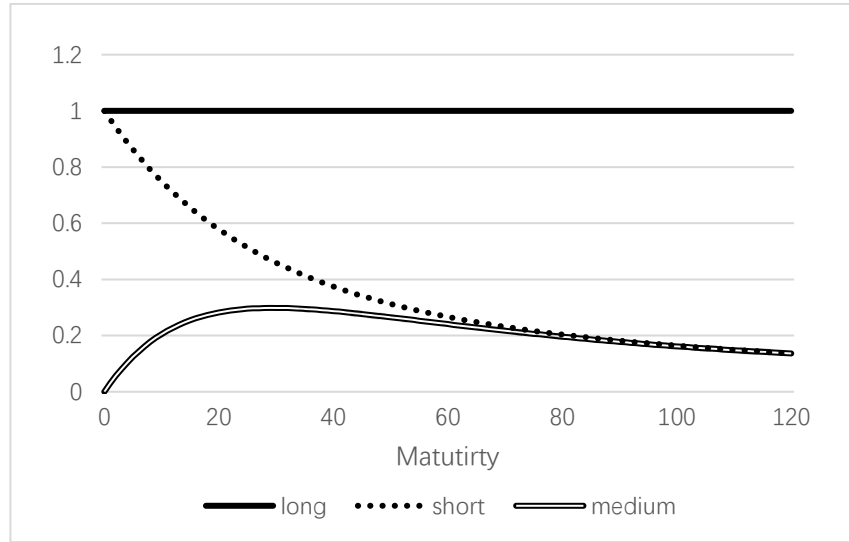
By inserting (5.1) into (5.2) and integrating, the corresponding zero-coupon yields with different maturities could be given as a function in terms of three unobserved factors at any point of time:

$$y(\tau) = \beta_1 + \beta_2 \left[\frac{1 - \exp(-\lambda \tau)}{\lambda \tau} \right] + \beta_3 \left[\frac{1 - \exp(-\lambda \tau)}{\lambda \tau} - \exp(-\lambda \tau) \right] \quad (5.3)$$

where $y(\tau)$ denotes yields and τ is maturity. Parameter λ is the exponential decay rate. Small (large) decay rate corresponding to slow (fast) decay fits better at the long (short) term yields. Parameters β_1 , β_2 and β_3 are interpreted as long-, short- and medium-term components on measuring the strengths of yield curve respectively. As shown in Figure IV-1, the loading on β_1 is constant 1 which does not decay to zero in limit. The loading on β_2 is a function $\frac{1 - \exp(-\lambda \tau)}{\lambda \tau}$ which starts at 1 and decays monotonically to zero. The loading on β_3 is a function $\frac{1 - \exp(-\lambda \tau)}{\lambda \tau} - \exp(-\lambda \tau)$ which

increases from zero at first and then decays to zero in the end. The three loadings give sufficient flexibility to the Nelson-Siegel model to generate different shapes of yield curve with various sets of weights of them.

Figure IV-1: Plots of Parameter Loadings



Note: λ is set as 0.0609 as Diebold and Li (2006).

The dynamic Nelson-Siegel model

Diebold and Li (2006) interpret the Nelson and Siegel (1987) as a dynamic latent factor model with time-varying $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$

$$y_t(\tau) = L_t + S_t \left[\frac{1 - \exp(-\lambda\tau)}{\lambda\tau} \right] + C_t \left[\frac{1 - \exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau) \right] \quad (5.4)$$

where the three latent factors are interpreted as level, slope and curvature factors of yield curve denoted as L_t , S_t and C_t respectively. The increase of L_t leads to an parallel shift of the whole yield curve and surely changes the level of the yield curve. Since S_t is the short rate component, the increase of it moves the short rate up and leads to a change on the slope of yield curve. An increase in the medium rate component C_t will load more on the medium-term rate than the short and long end of yield curve, thereby changing the curvature of the yield curve. This dynamic Nelson-Siegel

interpretation enable us to estimate the entire yield curve at any point of time by using only three factors.

Diebold, Rudebusch & Aruoba (2006) place the dynamic Nelson-Siegel model into the state-space system with two equations

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \end{pmatrix} \quad (5.5)$$

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ \vdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} - e^{-\tau_1\lambda} \\ 1 & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} & \frac{1-e^{-\tau_2\lambda}}{\tau_2\lambda} - e^{-\tau_2\lambda} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} - e^{-\tau_N\lambda} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{N,t} \end{pmatrix} \quad (5.6)$$

where $t = 1, \dots, T$. The transition equation (5.5) describes the dynamics of the three latent factors and the measurement equation (5.6) governs the relationship between the unobservable factors and the observable yields at various maturities. This state-space representation of dynamic Nelson-Siegel model facilitates the convenience on estimation, extraction of latent factors and so on.

In notation of vector, this model could be written as

$$(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t \quad (5.7)$$

$$y_t = \Lambda(\lambda)f_t + \varepsilon_t \quad (5.8)$$

where

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right] \quad (5.9)$$

$$E(f_0 \eta_t') = 0 \quad (5.10)$$

$$E(f_0 \varepsilon_t') = 0 \quad (5.11)$$

In transition equation (5.7), the state vector is formed by the unobservable factors as $f_t = (L_t, S_t, C_t)'$ and follows a first order vector autoregressive process⁵. μ is the mean matrix (3×1) of the state vector. $A_{3 \times 3}$ is the transition matrix which contains 9 coefficients to be estimated. The transition disturbances η_t is a white noise and the covariance matrix $Q_{3 \times 3}$ of it is unrestricted which allows the shocks to the factors to be correlated. In measurement equation (5.8), y_t denotes the vector of yields with N time series at different maturities. $\Lambda_{N \times 3}$ is the measurement matrix and the elements in it are the Nelson-Siegel factor loadings at all the maturities. The measurement disturbance is a white noise and its covariance matrix $H_{N \times N}$ is assumed to be diagonal which implies that the deviations of yields form the yield curve at different maturities are uncorrelated. Due to the requirements of the estimation, the transition and measurement disturbances are assumed to be orthogonal to each other and also to the initial state vector f_0 .

The macro-finance Nelson-Siegel model

With the state-space form dynamic Nelson-Siegel interpretation, it is convenient to add in macroeconomic variables to the state vector to construct the macro-finance extension. Following the work from Diebold, Rudebusch & Aruoba (2006), we add two macro variables inflation and industrial production growth denoted by CPI_t and IP_t to the state variables. The state vector is defined as $f_t = (L_t, S_t, C_t, CPI_t, IP_t)'$. Thus, the state-space form macro-finance Nelson-Siegel model could be written as:

⁵ First order vector autoregressive process allows the model to form a state-space system. As stated by Diebold, Rudebusch & Aruoba (2006), although ARMA state vector dynamics of any order may be readily accommodated in state- space form, the VAR with only one order is employed for transparency and parsimony.

$$\begin{pmatrix} L_t - \mu_L \\ S_t - \mu_S \\ C_t - \mu_C \\ CPI_t - \mu_{CPI} \\ IP_t - \mu_{IP} \end{pmatrix} = \begin{pmatrix} a_{11} & \cdots & a_{15} \\ \vdots & \ddots & \vdots \\ a_{51} & \cdots & a_{55} \end{pmatrix} \begin{pmatrix} L_{t-1} - \mu_L \\ S_{t-1} - \mu_S \\ C_{t-1} - \mu_C \\ CPI_{t-1} - \mu_{CPI} \\ IP_{t-1} - \mu_{IP} \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \\ \eta_{5,t} \end{pmatrix} \quad (5.12)$$

$$\begin{pmatrix} y_t(\tau_1) \\ \vdots \\ y_t(\tau_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} & \frac{1-e^{-\tau_1\lambda}}{\tau_1\lambda} - e^{-\tau_1\lambda} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} & \frac{1-e^{-\tau_N\lambda}}{\tau_N\lambda} - e^{-\tau_N\lambda} & 0 & 0 \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \\ CPI_t \\ IP_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{N,t} \end{pmatrix} \quad (5.13)$$

where the dimensions of A, μ, Λ, η_t and Q increase to 5×5 , 5×1 , $N \times 5$, 5×1 and 5×5 . The transition equation is a VAR (1) of the factors including latent and observable ones. In measurement equation (5.13), the factor loadings of observable macroeconomic factors are all assumed to be zero which is consistent with the previous assumption that only three factors are needed to extract information from the yield curve. Since we are only interested in the effects between yield curve and macro variables, not the possible influences between inflation and industrial production growth, the order of the two macro variables in the VAR (1) is not important.

In order to investigate the possible interactions between the Chinese yield curve and the macroeconomy, the yields-only (dynamic Nelson-Siegel) model written in state-space form will be estimated as a benchmark to be compared with the macro-finance models.

IV.2.2. Estimation Methods

Generally, most of the literature follows two methods to estimate the state-space Nelson-Siegel framework which are a two-step approach and a one-step approach, depending on whether the transition and measurement equations are estimated independently or simultaneously.

Two-step approach

The conventional two-step estimation procedure of Diebold and Li (2006) is proposed to estimate the yields-only dynamic Nelson-Siegel model without macroeconomic variables. It is assumed that the three state variables follow an independent and first-order autoregressive process in the two-step estimation procedure. Facing the problem of estimating the parameters L_t , S_t , C_t and λ in the first step, the measurement equation is estimated using cross-sectional data, in which the factor loadings are calculated for each time period, given the value of the decay parameter λ and the maturity τ . Since the decay parameter is fixed at a prespecified value, the measurement equation becomes linear and the factors can be estimated using ordinary least squares by treating the factor loadings as the regressors. In this way, the time series of each estimated factors are obtained. In the second stage, the transition equation is estimated by using the obtained factors in the first step.

For the use of two-step approach on estimating macro-finance model, Kollar (2012) follows Diebold and Li (2006)'s method in the first step and include three macroeconomic variables in the transition equation and the estimated latent factors from the first step and the observable macroeconomic factors are used in the second step.

One-step approach

Although the two-step approach is simple for computation, it produces inefficient estimators, since the uncertainties of the factor loadings calculation and the signal extraction that is inherent to the first-step estimate is neglected in the second step. Diebold, Rudebusch & Aruoba (2006) presents the one-step Kalman filter approach to estimate the dynamic Nelson-Siegel and macro-finance Nelson-Siegel models. The one-step approach estimates the transition and measurement equations simultaneously by using maximum likelihood which provides consistent and efficient parameter estimations.

In the state-space model consisted of (5.12) and (5.13), the recursive algorithm Kalman filter provides an optimal estimate of the vector of latent factors f_t conditional on the

information set and knowledge of the parameters of the state space μ, A, Λ, Q, H . The Kalman filter recursion consists of three steps. Firstly, we define $f_{t|t-1}$ as the expectation of f_t conditional on the information up to time $t - 1$ with mean square error (MSE) matrix $P_{t|t-1}$. We could have the prediction state vector at time $t - 1$ as:

$$f_{t|t-1} = \mu + Af_{t-1|t-1} \quad (5.14)$$

$$P_{t|t-1} = AP_{t-1|t-1}A' + Q \quad (5.15)$$

At time t , when y_t is observed, the prediction error could be calculated as:

$$v_t = y_t - y_{t|t-1} = y_t - \Lambda(\lambda)f_{t|t-1} \quad (5.16)$$

with the prediction error covariance matrix:

$$F_t = \Lambda(\lambda)P_{t-1|t-1}\Lambda(\lambda)' + H \quad (5.17)$$

At last, using the prediction error and the Kalman gain defined as $K_t = P_{t|t-1}\Lambda' F_t^{-1}$, the estimates could be updated as:

$$f_{t|t} = \mu + f_{t|t-1} + K_tv_t \quad (5.18)$$

$$P_{t|t} = P_{t|t-1} - K_t\Lambda(\lambda)P_{t|t-1} \quad (5.19)$$

With the initial value $f_{1|0}$ and $P_{1|0}$, the Kalman filter is recursively performed from $t = 1$ to $t = T$. The unknown parameters μ, A, Λ, Q, H are estimated by maximizing the log likelihood function as below:

$$L(\psi) = -\frac{NT}{2}\ln(2\pi) - \frac{T}{2}\ln|F_t| - \frac{1}{2}\sum_{t=1}^T v_t' F_t^{-1} v_t \quad (5.20)$$

Due to the efficiency of the on-step approach and the benefits of using Kalman filter and maximum likelihood, we employ the one-step method to estimate both yields-only and macro-finance models in this research. The two-step method is also applied and the obtained parameter estimation results will be used as the initial values in the one-step

method. The drawback of the one-step method is that the number of parameters to be estimated is huge. Diebold, Rudebusch & Aruoba (2006) include three observable factors and the parameters to be estimated in their models are 36 in yields-only and 81 in macro-finance. Considering the relatively shorter sample period of our research, we only include two macroeconomic factors which are inflation and industrial production, to the macro-finance model to reduce the number of parameters to be estimated.

IV.3. Data Description

The Chinese government bond yields data from inter-bank market is used due to the high participation and activity of government bond trading in inter-bank market. As stated in the data source ChinaBond, the Chinese yield curve is constructed by using bootstrapping on the coupon bonds and Hermite interpolation is applied to smooth the yields. Monthly yields of Chinese inter-bank government bond sampled from March 2006 to April 2015 is chosen to generate the yield curve with 10 different maturities which are 6, 12, 24, 36, 60, 84, 120, 180, 240 and 360 months. The choices of earliest date of the sample period is due to the data availability. The end day of month data is used. The data sample includes 110 months and 1100 monthly observation of yields at 10 maturities.

The macroeconomic variables applied to the macro-finance model are inflation and real activity from March 2006 to April 2015. The month-on-month change in consumer price index is used to measure the inflation shock. The data is collected from the National Bureau of Statistics of China. Since GDP is only reported quarterly, the interpolating low frequency data to high frequency may lose important information contained in the original data. In order to save information by using variable with same frequency, the seasonal adjusted industrial production is employed to capture the real activity in China. The growth rate of industrial production is used and the data source

is the World Bank⁶.

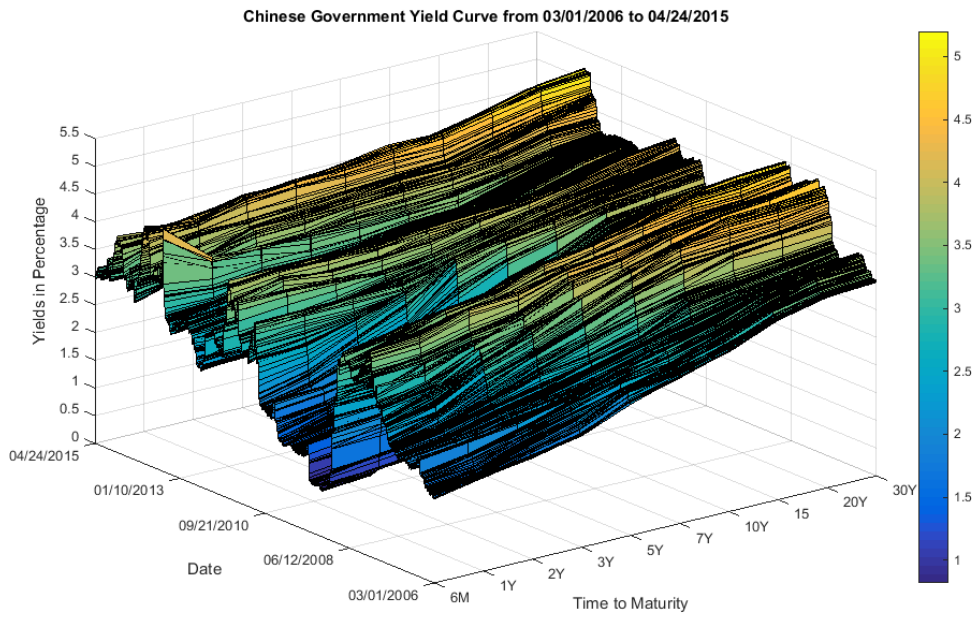
As shown below in Figure IV-2 and Figure IV-3, the Chinese yield curve from March 2006 to April 2015 is mostly upward sloping with no complex shape at each time point and dramatically declines on the interest rates with all maturities from September 2008 to December 2008. Also the term structure of interest rates in China exhibits a mean-reverting tendency.

Specifically, distinctive increases of the interest rates occurred from the beginning of 2006 till the end of 2007. The reason is that the rising of the stock market over the past two years leads to a decline in stock demand. At the same time, the central bank increased the required reserve ratio 18 times consecutively before September 2008 to reduce the surplus liquidity and keep the high inflation down. In August of 2008, the entire yield curve plunged due to the bankrupt of Lehman Brothers. The People's Bank of China cut the required reserve ratio three times to ensure reasonably adequate liquidity in the banking system and keep the economy humming. Also the Chinese government announced the 4 trillion yuan (\$586 billion) stimulus program⁷ on November 9, 2008 to boost the domestic consumption and offsetting the drawbacks from slowing down export caused by the world economic downturn. From 2009 to 2012, continuous fluctuation and slight rising of the interest rates at the latter half of year 2010 is observed and the interest rates overall level is trending up. Both in June and December of 2013, Chinese market was roiled by the unprecedented liquidity squeeze across banks. Interest rates with all maturities raised dramatically within weeks.

⁶ Since the National Bureau of Statistics of China only provide value-added of industry, the data we employed is the industrial production calculated by the World Bank using the value-added of industry data.

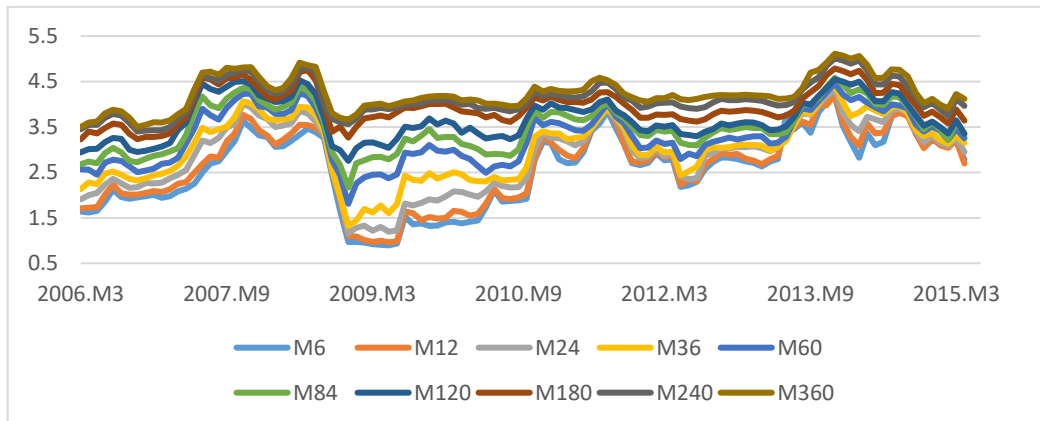
⁷ This stimulus package includes 10 major steps to spark growth as fiscal and monetary policies ease, which finances the areas of housing, rural infrastructure, transportation, health and education, environment, industry, disaster rebuilding, incomes, taxes and finance.

Figure IV-2: Chinese Government Yield Curve (03/01/2006-04/24/2015)



Note: Maturities from 6, 12, 24, 36, 60, 84, 120, 180, 240 and 360 Months

Figure IV-3: Chinese Spot Yields (03/01/2006-04/24/2015)



Note: Maturities from 6, 12, 24, 36, 60, 84, 120, 180, 240 and 360 Months

In order to extract information from the yields data, we provide the descriptive statistics of the Chinese yields data displayed in Table IV-1 which includes mean, standard deviation, minimum, maximum, skewness, kurtosis and autocorrelations at 1, 100 and 200 displacements. The results show some interesting characteristics. Firstly, the fact that as the maturity increase the mean of the yields rises which illustrates that the yields

curve is upward sloping. Secondly, the decreasing of the standard deviation of yields by maturity reveals that the short rates are much more volatile than the long rates. Then, the skewness moves upward from negative to positive value around zero and the kurtosis also has an uptrend which approaches zero. At last, according to the autocorrelation, the persistence of the yields is high and yields with shorter maturities are relatively more persistent than yields with longer maturities.

Table IV-1: Descriptive Statistics of Chinses Treasury Yields

Month	Mean	Std.	Min	Max	Skew.	Kur.	$\rho(1)$	$\rho(6)$	$\rho(12)$
6	2.554	0.836	0.818	4.374	-0.282	-0.857	0.998	0.618	0.272
12	2.653	0.816	0.887	4.250	-0.347	-0.852	0.999	0.616	0.246
24	2.861	0.781	1.096	4.418	-0.308	-0.795	0.999	0.613	0.186
36	3.029	0.692	1.244	4.500	-0.206	-0.722	0.998	0.552	0.108
60	3.290	0.587	1.773	4.529	0.014	-0.880	0.998	0.484	0.019
84	3.496	0.532	2.122	4.670	0.057	-0.837	0.998	0.445	-0.020
120	3.678	0.470	2.671	4.722	0.273	-0.899	0.997	0.407	-0.098
180	3.952	0.407	3.196	4.909	0.363	-0.525	0.997	0.323	-0.092
240	4.123	0.391	3.378	5.097	0.387	-0.286	0.998	0.347	-0.048
360	4.237	0.387	3.480	5.199	0.347	-0.351	0.996	0.360	-0.028

We conduct the Principle Components Analysis on the treasury yields of China. As shown in Table IV-2, the result shows that about 88.81% of the variation in China's Treasury bond yield is able to be explained by the first component and almost 99.44% of the movements in China's T-bond yield are explained by the first three components cumulatively.

Table IV-2: Principle Component Analysis of Chinese Treasury Yields

Component	Value	Difference	Proportion	Cumulative Proportion
1	8.8810	7.9157	0.8881	0.8881
2	0.9652	0.8679	0.0965	0.9846

3	0.0972	0.0791	0.0097	0.9944
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IV.4. Empirical Results

Before processing the one-step maximum likelihood estimation approach, we estimate the model by using the Diebold –Li two-step OLS method and the estimation results are used as the guess value of parameters required in the one-step approach. Also λ is prespecified as 0.2989 at which the loading of curvature is maximized in two-step estimation and the 0.2989 is set as the starting value of λ in the one-step method as well.

IV.4.1. Parameter Estimation

We have 56 free parameters to be estimated in the macro-finance model. In the transition equation, there are five parameters in the mean state vector $\mu_{5 \times 1}$ and 25 in the transition matrix $A_{5 \times 5}$. In the measurement equation, the measurement matrix contains one parameter λ . In addition, the unrestricted transition disturbance covariance matrix Q consists of 15 free parameters which are the five disturbance variance for the three latent factors and two macroeconomic variables, and ten covariance terms of them. The measurement disturbance matrix H which is assumed to be diagonal contains 10 parameters which are the disturbance variance for each of the yields with different maturities. The yields-only model contains 29 parameters.

Table IV-3 presents the parameter estimation of transition matrix A including macro-finance model in panel 1 and yields-only model in panel 2. The diagonal elements show the estimated coefficients of the three latent factors and two macroeconomic variables with their own lagged terms and all of them are statistically significant except for the CPI_t in macro-finance model. And the result indicates high persistence of own dynamics of L_t , S_t and C_t which are 0.864, 0.929 and 0.736 in macro-finance model and 0.864, 0.939 and 0.750 in yields-only model. But not as the U.S. data which has a decreasing persistence in yield curve factors indicated in Diebold, Rudebusch & Aruoba (2006), the second component slope has the highest dynamic persistence in China. The last column displays the estimates of the factor means μ which are all statistically

significant in both models. Also, half of the off-diagonal estimates are significant and the result indicates interesting findings.

For the yield factor interaction, positive L_{t-1} on C_t and S_{t-1} on L_t effects are found in both models. In addition, the positive effect of lagged level and negative effect of lagged curvature on slope factor are also statistically significant in macro-finance model. From the upper right block of matrix A in macro-finance model, the lagged industrial production growth rate is negatively related to the level factor while positively related to the curvature factor, although the relationship is small in magnitude but strong in significance. The yields effect on macroeconomy shows that the lagged curvature factor influences the inflation rate positively. Furthermore, the result which shows that the lagged slope factor is positively related to the movements of industrial production is consistent with the fact that the decrease of the slope of the yield curve (increase of the slope factor) always indicates slowdown of economy, for the industrial production represents the economy activity.

Table IV-3: Estimates of Matrix A and Factor Means

	L_{t-1}	S_{t-1}	C_{t-1}	CPI_{t-1}	IP_{t-1}	μ
Panel 1: Macro-Finance Model						
L_t	0.864*** (0.045)	0.155** (0.072)	-0.009 (0.140)	0.104 (0.151)	-0.061*** (0.022)	4.491*** (0.107)
S_t	0.047** (0.022)	0.929*** (0.032)	-0.155** (0.068)	0.038 (0.066)	-0.008 (0.009)	-2.260*** (0.211)
C_t	0.063*** (0.020)	0.016 (0.030)	0.736*** (0.060)	-0.040 (0.060)	0.037*** (0.009)	-0.372** (0.187)
CPI_t	-0.038 (0.024)	0.003 (0.044)	0.216** (0.089)	0.133 (0.092)	0.008 (0.012)	0.233*** (0.077)
IP_t	-0.029 (0.054)	0.164* (0.087)	-0.110 (0.171)	0.513*** (0.178)	0.890*** (0.024)	0.978*** (0.109)
Panel 2: Yields-Only Model						
L_t	0.864***	0.117*	-0.018			4.519***

	(0.035)	(0.066)	(0.066)	(0.107)
S_t	0.038*	0.939***	-0.131	-2.126***
	(0.020)	(0.037)	(0.081)	(0.319)
C_t	0.055***	0.042	0.750***	-0.448*
	(0.017)	(0.033)	(0.063)	(0.254)

Notes: The asterisks *, ** and *** represent the 10%, 5% and 1% statistical significance levels respectively and the estimation standard errors are indicated in the parentheses.

The parameter estimates of the factor covariance matrix Q are presented in Table IV-4 for both models. All the diagonal elements in the Q matrix show individual significance and the estimates are 0.020, 0.081, 0.306, 0.312 and 0.006 in macro-finance model and 0.020, 0.083 and 0.330 in yields-only model respectively, and this result illustrates that the transition shock volatility is increasing from the level factor to slope and to curvature. This feature is consistent with the finding in the U.S. Two out of ten and two out of three off-diagonal terms in macro-finance and yields-only models are significant. Since we assume Q is a full matrix in our models, the likelihood ratio and Wald test are both applied to examine the joint significance of the off-diagonal terms in covariance matrix. The test results of both models reject the null-hypothesis of the diagonality of the covariance matrix as given in the panel *c* of Table IV-4.

Table IV-4: Estimates of Covariance Matrix

	a. Macro-Finance Model					b. Yields-Only Model		
	$\eta_{1,t}$	$\eta_{2,t}$	$\eta_{3,t}$	$\eta_{4,t}$	$\eta_{5,t}$	$\eta_{1,t}$	$\eta_{2,t}$	$\eta_{3,t}$
$\eta_{1,t}$	0.020*** (0.003)	-0.005 (0.004)	-0.016 (0.010)	-0.010 (0.008)	-0.001 (0.001)	0.020*** (0.003)	-0.006 (0.005)	-0.020* (0.012)
$\eta_{2,t}$		0.081*** (0.011)	-0.055*** (0.016)	0.018 (0.015)	-0.002 (0.002)		0.083*** (0.012)	-0.055*** (0.018)
$\eta_{3,t}$			0.306*** (0.051)	0.043 (0.029)	0.008** (0.004)			0.330*** (0.054)
$\eta_{4,t}$				0.312*** (0.041)	0.005 (0.004)			
$\eta_{5,t}$					0.006***			

(0.001)						
c. Tests for diagonality of covariance matrix Q						
Macro-Finance Model				Yields-Only Model		
	Test Statistics	DF	P-Value	Test	DF	P-Value
Likelihood	31.765	10	0.000	23.158	3	0.000
Wald	35.796	10	0.000	23.707	3	0.000

Notes: The asterisks *, ** and *** represent the 10%, 5% and 1% statistical significance levels respectively and the estimation standard errors are indicated in the parentheses.

The descriptive statistics of measurement errors of yields at selected maturities in this research are displayed in Table IV-5. The mean and standard deviation of residuals from both macro-finance and yields-only models are very close and yet small which indicates good fit of both models to the Chinese government yields data.

Table IV-5: Summary Statistics for Measurement Errors of Yields

Maturity (Month)	Macro-finance model		Yields-only model	
	Mean	Std. Dev.	Mean	Std. Dev.
6	0.001	0.292	0.004	0.289
12	-0.012	0.269	-0.006	0.269
24	0.003	0.251	0.011	0.254
36	0.004	0.236	0.013	0.237
60	-0.005	0.209	0.005	0.208
84	-0.005	0.207	0.006	0.205
120	-0.053	0.197	-0.041	0.196
180	-0.016	0.175	-0.005	0.173
240	0.016	0.162	0.026	0.159
360	-0.024	0.161	-0.015	0.157

IV.4.2. Extraction of Latent Factors

We extract the optimal latent factors level, slope and curvature by applying the Kalman smooth algorithm for both yields-only and macro-finance models. The descriptive statistics of the extraction of level, slope and curvature are displayed in Table IV-6.

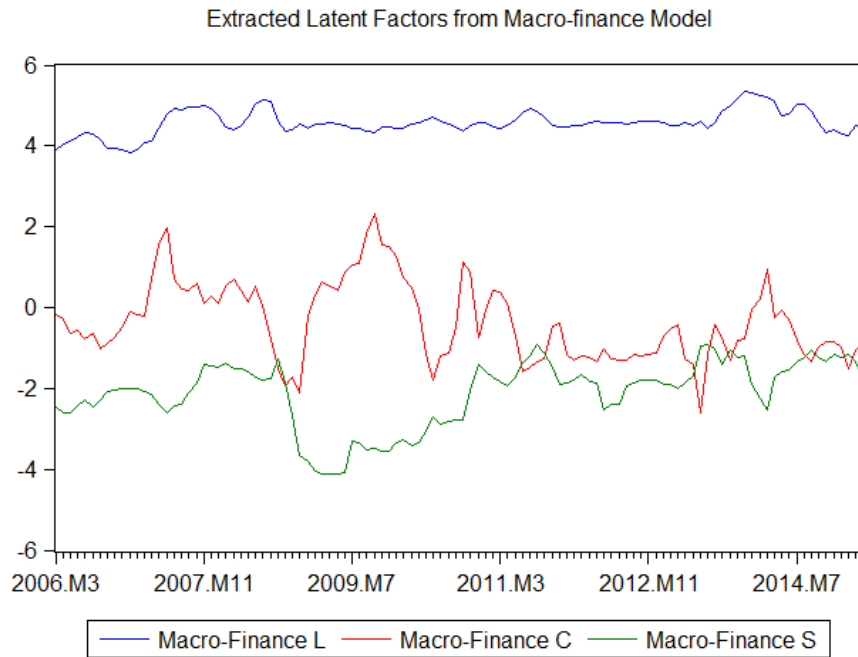
According to the results, we find that the latent factors extracted from yields-only models are quite close in magnitude to those extracted from macro-finance models. The autocorrelation result suggests that the slope is the most persistent factor among the three.

Table IV-6: Descriptive Statistics of the Extraction of Latent Factors

	Macro-Finance Model			Yields-Only Model		
	\hat{L}_t	\hat{S}_t	\hat{C}_t	\hat{L}_t	\hat{S}_t	\hat{C}_t
Mean	4.571	-2.121	-0.362	4.570	-2.120	-0.361
Std. Dev.	0.308	0.821	0.960	0.309	0.802	0.959
Minimum	3.846	-4.120	-2.604	3.870	-4.118	-2.605
Maximum	5.344	-0.910	2.309	5.339	-0.913	2.286
Skewness	0.148	-0.825	0.470	0.169	-0.825	0.475
Kurtosis	3.288	2.896	2.763	3.245	2.898	2.754
$\rho(1)$	0.882	0.937	0.830	0.882	0.936	0.825
$\rho(6)$	0.209	0.516	0.167	0.208	0.515	0.173
$\rho(12)$	-0.094	0.136	0.060	-0.101	0.136	0.072

According to Figure IV-4, the estimated level factor is positive during the whole sample period and fluctuates around 4.5 percent; the estimated slope factor is negative and moves around -2 percent and the estimated curvature factor takes both positive and negative values and fluctuates around zero percent. These findings are consistent with the results from other studies based on the U.S. data.

Figure IV-4: Extractors of Latent Factors from Macro-Finance Model



In Figure IV-5, Figure IV-6 and Figure IV-7, we plot each of the extracted yield curve factors from yields-only and macro-finance models with its respective data-based counterpart. The empirical yields factors are defined according to popular studies. The data-based level is the 360-month yields, slope is the difference between the 6-month and 360-month yields, and curvature is two times the 60-month yields minus the sum of the 360-month and 6-month yields.

Figure IV-5: Extracted level factors from yields-only and macro-finance models and data-based counterpart

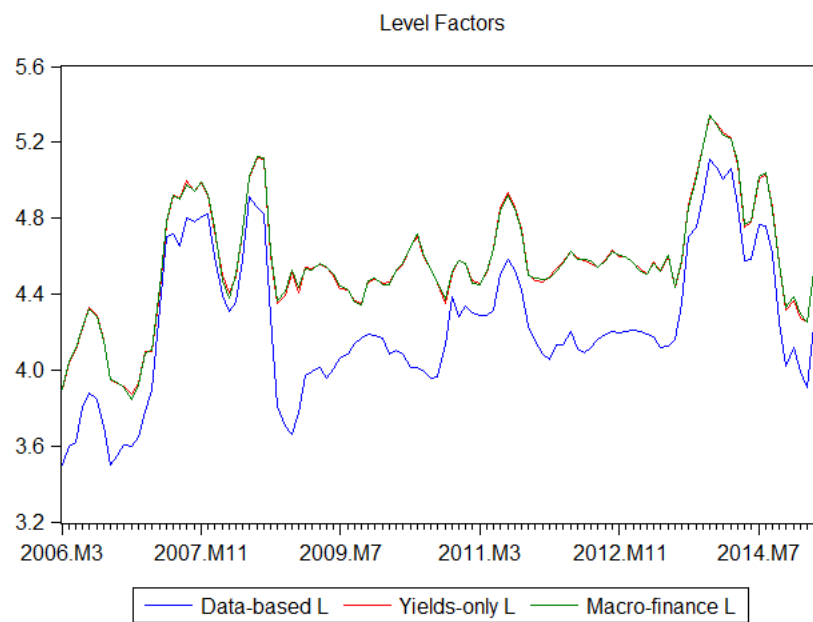


Figure IV-6: Extracted slope factors from yields-only and macro-finance models and data-based counterpart

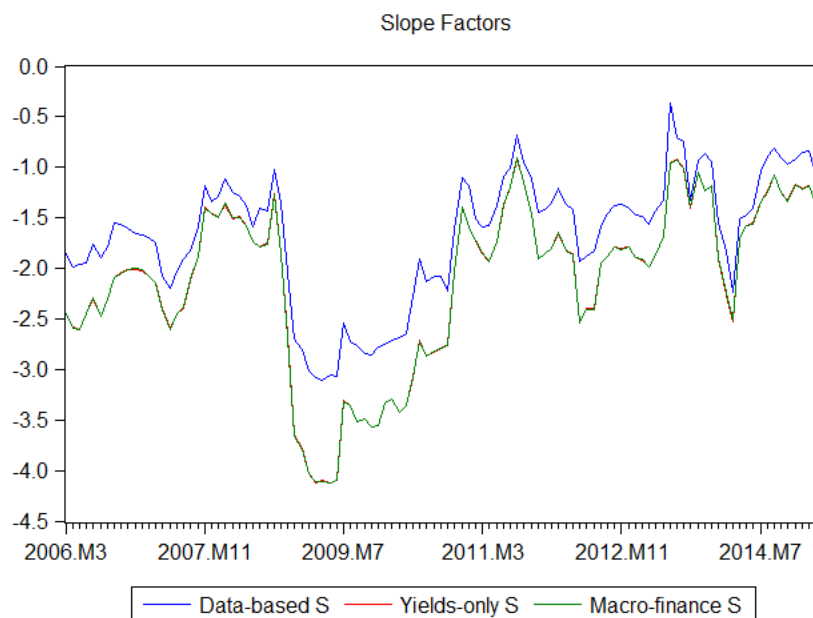
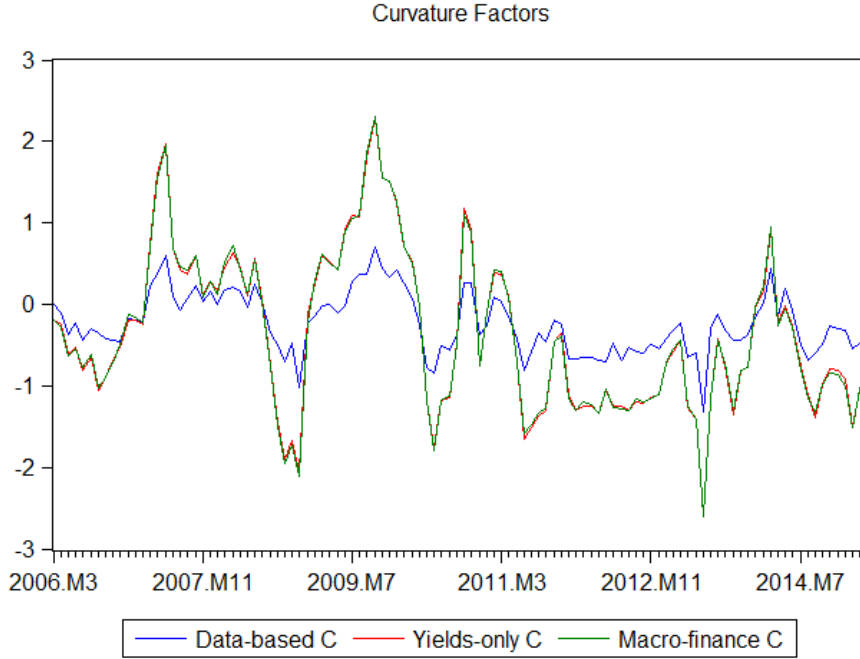


Figure IV-7: Extracted curvature factors from yields-only and macro-finance models and data-based counterpart



The Kalman smoothed factors from both yields-only and macro-finance models are very close to their empirical counterparts. The correlations between the estimated factors and their respective empirical counterparts ⁸ are $\rho(L_t, \tilde{L}_t) = 0.924$, $\rho(S_t, \tilde{S}_t) = 0.986$ and $\rho(C_t, \tilde{C}_t) = 0.952$ in macro-finance model and $\rho(L_t, \tilde{L}_t) = 0.927$, $\rho(S_t, \tilde{S}_t) = 0.986$ and $\rho(C_t, \tilde{C}_t) = 0.952$ in the yields-only model. The high correlation between estimated and empirical latent factors indicates that the yields curve is well represented by the level, slope and curvature factors in both models.

IV.4.3. Impulse Response Function

The impulse response functions are employed to investigate the dynamic system of the

⁸ The data-based factors are denoted as \tilde{L}_t , \tilde{S}_t and \tilde{C}_t .

yield curve and economy. The results are given in Figure IV-8 with 90 percent confidence intervals and categorized to four groups for interpretation.

The first group is the yield curve responses to yield curve shocks as shown in the upper left 3×3 block of Figure IV-8. The own effects of the three latent factors are all significant and the slope factor shows the highest persistence. Most of the off-diagonal responses of yield curve to yield curve shocks are insignificant. Interestingly, the increase of curvature factor increases the level of the yield curve and the increase of the slope factor (decrease of slope of yield curve) reduces the curvature of yield curve.

The upper-right 3×2 block of Figure IV-8 exhibits the responses of yield curve to the macroeconomic shocks. Unfortunately, the responses of level and slope factors to the macroeconomic shocks are insignificant. However, it is unusual that the curvature of the yield curve increases in response of the higher inflation. This result is quite different from the other studies based on US. In most US studies, both the inflation and real activity have little impact on the curvature factor while both level and slope factors respond directly to shocks in macro variables.

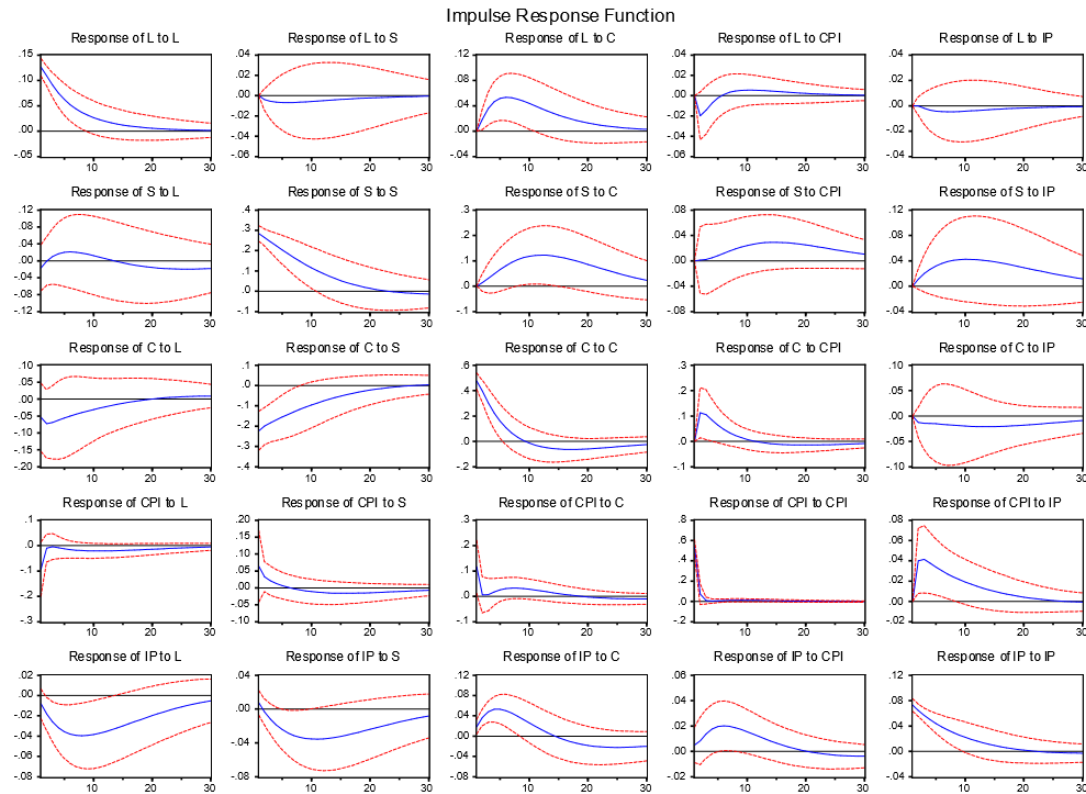
The macroeconomic responses to the yield curve shocks are given in the left bottom 2×3 block of Figure IV-8. The responses of inflation level to all the yield curve shocks are weak and insignificant. The industrial production growth declines in responses of the increases of level and slope factors of yield curve and increases with the curvature, but all the responses are in very small scales. In contrast, the U.S. data shows negligible responses of macro variables to the shock in curvature factor, but all the macroeconomic variables increase to positive shocks in level factor.

The last category is the macroeconomic responses to macroeconomic shocks. Both diagonal responses are significant but the inflation's own dynamic effect is relatively strong in magnitude and less consistent than the industrial production. In addition, the inflation level increase is associated with the rise of industrial production.

In all, the close relationship between the level factor and inflation found in the U.S.,

seems insignificant in China.

Figure IV-8: Impulse Response Functions of Yield Factors and the Macro Variables



IV.4.4. Variance Decomposition

The variance decomposition technique is employed as well to explore the possible interaction between yields curve and macroeconomic variables. Table IV-7 and Table IV-8 display the variance decomposition results of the yield curve latent factors and two additional macroeconomic factors at 10 different forecast horizons from 1 up to 10 months respectively.

As shown in the first panel of Table IV-7, the movement of level factor is totally explained by its own at the one-month horizon and mostly determined by itself even at longer forecast horizons. Unfortunately, the inflation rate and industrial production take negligible effects on the variation in level factor.

In the second panel of Table IV-7, the level factor accounts for an increasing amount on the variation in slope factor. At the longest horizon 10 months, the level factor contributes 15 percent of movements on slope factor. The industrial production also accounts for a remarkable participation in explaining the movement of slope factor. 10 to 23 percent of dynamics of slope factor is driven by industrial production movements at forecast horizons longer than 5 months up to 10 months.

The variance decomposition of the curvature factor is given in the third panel of Table IV-7. The curvature factor movement is totally explained by its own effect at the one month's forecast horizon. As the forecast horizon moves longer, the level and slope factors account for increasing proportions of the curvature factor variation, but the slope factor of 26% shows more explanatory power in curvature variation than the level factor of 3% at 10 months forecast horizon. Interestingly, the inflation effect of curvature factor has more than 4 percent at medium-term horizons and decreases at longer periods. Also the industrial production effect on curvature is increasing as the horizon moves longer and approaches the maximum of 5 percent in explaining the curvature variation.

Table IV-7: Variance Decomposition of Latent Factors with Monthly Data

Variance Decomposition of L_t					
Horizon	L_t	S_t	C_t	CPI_t	IP_t
1	100.00	0.000	0.000	0.000	0.000
2	99.519	0.125	0.223	0.084	0.048
3	98.736	0.323	0.607	0.090	0.244
4	97.821	0.540	1.030	0.079	0.530
5	96.915	0.751	1.428	0.069	0.837
6	96.091	0.943	1.775	0.064	1.127
7	95.375	1.114	2.068	0.061	1.383
8	94.768	1.265	2.309	0.060	1.598
9	94.262	1.396	2.506	0.059	1.776
10	93.843	1.511	2.667	0.059	1.920
Variance Decomposition of S_t					
Horizon	L_t	S_t	C_t	CPI_t	IP_t

1	0.000	100.00	0.000	0.000	0.000
2	1.261	97.318	0.013	0.000	1.408
3	3.409	92.510	0.072	0.000	4.009
4	5.793	86.910	0.193	0.001	7.103
5	8.054	81.311	0.376	0.002	10.258
6	10.034	76.096	0.607	0.005	13.259
7	11.692	71.413	0.869	0.009	16.017
8	13.039	67.287	1.146	0.016	18.512
9	14.108	63.691	1.427	0.024	20.750
10	14.938	60.573	1.702	0.033	22.754
Variance Decomposition of C_t					
Horizon	L_t	S_t	C_t	CPI_t	IP_t
1	0.000	0.000	100.00	0.000	0.000
2	0.005	1.470	94.915	2.867	0.743
3	0.008	4.272	90.628	4.058	1.033
4	0.038	7.891	86.403	4.429	1.238
5	0.153	11.828	82.053	4.458	1.509
6	0.416	15.669	77.653	4.338	1.924
7	0.861	19.129	73.326	4.155	2.529
8	1.481	22.048	69.186	3.949	3.336
9	2.237	24.376	65.318	3.741	4.327
10	3.076	26.135	61.782	3.543	5.465

It can be concluded that the macroeconomic variables have significant impacts on the movement of yield curve. The industrial production has a remarkable influence on the variation in slope factor of yield curve. The curvature factor could be explained by both inflation and industrial production at medium and longer horizons. However, the macroeconomic effects on the level factor of yield curve is negligible.

Table IV-8 provides the variance decomposition of the two macroeconomic variables inflation and industrial production. As the first panel shown, 100 percent of the variation in inflation rate is driven by its own at the 1-month horizon, when moves to

longer horizons, it is still largely determined by the variations in industrial production which accounts for 20 percent at 2-month horizon and reaches almost 60 percent at 10-month horizon. The yield curve factor effects on the inflation rate are negligible. The entire industrial production variation is determined by its own movement at 1-month forecast horizon. Slope factor, curvature factor and inflation influence on the industrial production is insignificant. However, the level factor explains more than 6 percent of the industrial production movement at 10 months forecast horizon.

Table IV-8: Variance Decomposition of Macroeconomic Variables for Monthly Data

Variance Decomposition of CPI_t					
Horizon	L_t	S_t	C_t	CPI_t	IP_t
1	0.000	0.000	0.000	100.000	0.000
2	0.837	0.110	0.122	78.604	20.328
3	1.073	0.229	0.104	64.139	34.456
4	1.038	0.307	0.109	55.728	42.818
5	0.951	0.344	0.160	50.407	48.139
6	0.885	0.349	0.234	46.775	51.757
7	0.863	0.339	0.312	44.161	54.325
8	0.893	0.324	0.381	42.210	56.193
9	0.972	0.315	0.437	40.713	57.562
10	1.100	0.320	0.479	39.541	58.560
Variance Decomposition of IP_t					
Horizon	L_t	S_t	C_t	CPI_t	IP_t
1	0.000	0.000	0.000	0.000	100.000
2	0.206	0.004	0.076	0.003	99.711
3	0.626	0.025	0.183	0.016	99.150
4	1.208	0.078	0.281	0.033	98.399
5	1.915	0.173	0.356	0.051	97.505
6	2.721	0.314	0.404	0.067	96.495
7	3.605	0.502	0.428	0.080	95.384
8	4.550	0.734	0.435	0.090	94.191
9	5.539	1.004	0.430	0.098	92.929

10	6.557	1.304	0.418	0.103	91.618
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The results of macroeconomic variable variance decompositions indicate that the yields-to-macro effect exists in China. The level factor of yield curve has significant impacts on the industrial production at longer forecasting horizons. But the yields effects on inflation is weak.

In short, there is a bidirectional influence between the Chinese yield curve movements and the macroeconomic variables. The macroeconomy effect on the yield curve enters at medium-term horizons while the yields influence on the macroeconomy is only significant at the long-term horizons and relatively weak. In contrast, Diebold, Rudebusch & Aruoba (2006) report similar finding with us with evidence from the U.S. They conclude that the macroeconomic effects on future yield curve is stronger than the yield curve effects on the future macroeconomy.

IV.5. Summary and Conclusion

Term structure models have been investigated intensively within the U.S. and other mature markets. In contrast, study in modelling interest rates in emerging markets has not yet reached a consensus. Due to low liquidity at certain regions of the yield curve and a long-time controlling by the government in the past, in-depth study of Chinese government bond markets is urgent, especially on the interaction between the yield curve and the macroeconomy.

This paper employs a dynamic Nelson-Siegel macro-finance model to examine the possible interactions between movement on Chinese yield curve and the macroeconomy. The macroeconomic variables used in this study are inflation and real activity which are measured by CPI and industrial production. The model is estimated by one-step maximum likelihood approach via Kalman filter instead of the conventional two-step ordinary least squares regression method. Following Diebold, Rudebusch & Aruoba (2006), we write the macro-finance model in state-space form. This system enables us

to extract latent factor directly and apply impulse response function and variance decomposition to analyze the interactions between the term structure of interest rates and macroeconomy.

Interestingly, bidirectional causality is found, but the yield curve effects on the macroeconomy is relatively weak than the reverse influences. In the long-term horizon, the inflation and real activity can explain more than 30 percent of the variation of yield curve. These results indicate flexibility and capacity of the dynamic Nelson-Siegel macro-finance model on describing the yield curve of emerging market. These results indicate flexibility and capacity of the dynamic Nelson-Siegel macro-finance model on describing the yield curve of emerging market.

To sum up, this study establishes that the slope and curvature of the yield curve are influenced by the state of the economy evidence that the long end of the curve reflects the market's view regarding the economy, whilst the short end behaves very much independently of such long-term views, simply reflecting the policy decisions. Possible suggestion for further research is to impose the condition of no-arbitrage opportunity to the macro-finance model.

Chapter V. Regime-Switching Diffusion Models of Short-Term Interest Rate

V.1. Introduction

The risk-free short-term interest rate plays a fundamental role in financial economics. First, the default-free short rate composes the short end of the yield curve, therefore the pricing of fixed-income securities and derivatives at all the maturities are associated with it. Second, the short rate is the reference rate for asset pricing in terms of excess returns. Third, the short rate is the main instrument of monetary policy. Thus, studies on modelling the dynamics of the short-term interest rate are of considerable interest.

The earlier models of short rate are single factor diffusion models, such as Vasicek (1977), Cox, Ingersoll and Ross (1985) (hereafter CIR), Chan, Karolyi, Longstaff and Sanders (1992) (hereafter CKLS). Although these one-factor diffusion models can describe the mean-reverting tendency and level effect of the short rate well, they are unable to explain the phenomenon of volatility clustering and leptokurtosis of short rate. Consequently, Brenner, Harjes and Kroner (1996) incorporated the GARCH effect to the drift of diffusion models. Andersen and Lund (1997) and Ball and Torous (1999) introduced the stochastic volatility to the CIR and CKLS models respectively. Their studies showed that both GARCH effects and stochastic volatility can largely improve the in-sample fitting of diffusion models. However, the GARCH-based models are found to suffer from the explosive volatility problem.

These models all assume that there exists only one regime for the conditional mean and variance of the short rate. However, based on the evidence from the U.S., the short rate indicates higher volatility during the OPEC oil crisis (1974 and 1979), Federal Reserve Monetary Experiment (1979-1982) and the stock market crash (1987). Many in the literature (Hamilton (1988), Cai (1994), Gray (1996), Garcia and Perron (1996), etc.) claim the existence of regime changes and suggest the use of regime-switching models instead of the single-regime models, due to the diverse behaviour of the short rate in various economic environment. The regime-shifting models are capable of modelling

the short rates which are produced under each set of economic circumstances. In addition, regime shifting can improve capturing the volatility clustering and avoid the explosive volatility issue that came up in the diffusion models with GARCH effect and stochastic volatility.

Comparing with the fruitful research in advanced economies especially the U.S., there are relatively fewer studies of short-rate dynamics in emerging markets, especially in China. However, as the Chinese government bond market is more market-oriented recent years, more and more research has been attempted to capture the movements of short-term interest rates in China. Xie and Wu (2002) examine the dynamic behaviour of one-month Chinese Interbank Offered Rate (hereafter CHIBOR) from 1996 to 1999 by using Generalized Method of Moments estimation method and argue that Vasicek model fits the data better than the CIR model. Chen and Xie (2004) compare the Vasicek, CKLS and GARCH models with and without regime-switching assumption by following the method from Gray (1996) and finds the existence of two regimes in the dynamics of three-month treasury yield. Hong and Lin (2006) test a wide variety of short rate models by using the 7-day repurchasing rate from the Shanghai Stock Exchange market and suggest that the introduction of GARCH, regime-switching models and jump effects can largely improve the fitting degree of one-factor diffusion models. These studies have provided pioneer works on modelling the Chinese short rate dynamics and many works suggest the use of regime-switching models to capture the structure changes occurred in Chinese short rate. However, the data source of Chinese short-term interest rate in these studies varies a lot and most are outdated, as a result of the relatively short history of Chinese government bond market.

Hence, in this study we employ up-to-date three-month yields data from inter-bank government bond market, and try to compare the ability of different short rate models to capture the actual behaviour of Chinese short rate in one framework. Within one framework, it is easier to evaluate relative performance of each model in a consistent way. The framework we employed is Conley, Hansen, Luttmer and Scheinkman (1997) (hereafter CHLS) and the other nested models compared in this study are Vasicek (1977), CIR (1985) and CKLS (1992) short rate models. In addition, corresponding

Markov switching two-regime extensions of each model are compared to capture the possible regime-shifting behaviour. All these models are nested in the CHLS (1997) framework by imposing different restrictions on parameters and the generalized form short-rate model allows us to test level effect of short rate on diffusion term and nonlinear restriction on drift term in one system.

The rest of this chapter is organized as follows. Section 2 describes the data, the short rate models employed and the estimation method. The empirical analysis is presented in section 3, including interpretation of estimation results and model comparison. The last section 4, gives the conclusion.

V.2. Data and Model

V.2.1. Data Description

Since Chinese government bond market is relatively young, majority studies in literature apply different proxies to measure Chinese risk-free short rate, such as one-month CHIBOR, 7-day repo rate in stock exchange and 7-day or one-month SHIBOR. Throughout the two decades' development, although the segmentation of market still exists, the inter-bank market dominates the whole secondary market by absorbing most of the transactions. Meanwhile, the liquidity of the Chinese government bond has increased dramatically. Thus, the inter-bank government bond yields are able to reflect the market rates.

In this chapter, the three-month inter-bank treasury yield is employed, considering the liquidity of treasury bonds with maturity less than 3-month is relatively low. The data set collected from ChinaBond, includes 485 weekly observations sampled from 1st March, 2006 to 30th December, 2015. Wednesday's rates are used and all the interest rates are annualized at a 360-day year base.

Table V-1 Summary statistics of 3-month treasury yields

Mean	Std.	Skewne	Kurtosis	JB	JB <i>p</i> -	Correlation
------	------	--------	----------	----	---------------	-------------

R	2.4751	0.8409	-0.1456	2.3450	10.3834	0.0056	-0.0926
ΔR	0.0016	0.1429	0.6248	16.9990	3983.59	0.0000	

Table V-1 displays the descriptive statistics of the three-month government yield and its first difference. As shown in the first column, the mean of change in three-month yield is close to zero. The kurtosis of the difference is much larger than three times the standard deviations, indicating the existence of ‘extreme’ movements and the Jarque-Bera statistics reject the normality. The negative correlation coefficient between level and its first difference indicates mean reversion tendency on the movement of short rate. The histogram of changes on short rate is displayed in Figure V-1. The peak of the histogram is higher than that of normal distribution and the data are skewed right slightly. Both Figure V-1 and Figure V-2 indicate fatter tail on the change of interest rate compared to the normal distribution.

Figure V-1: Histogram of change on Chinese 3-month treasury yields

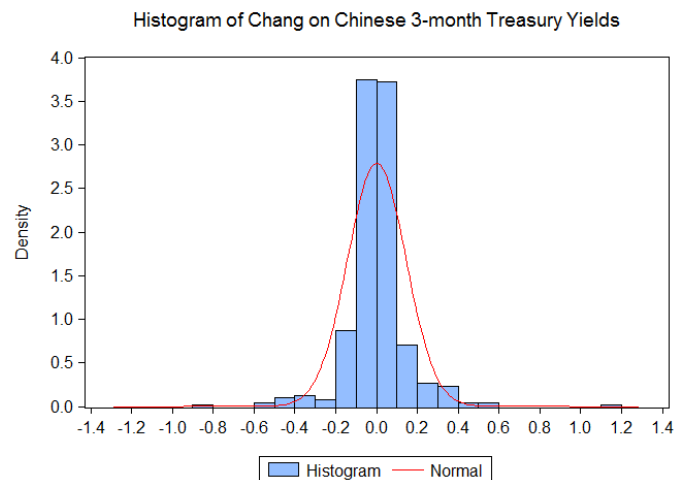
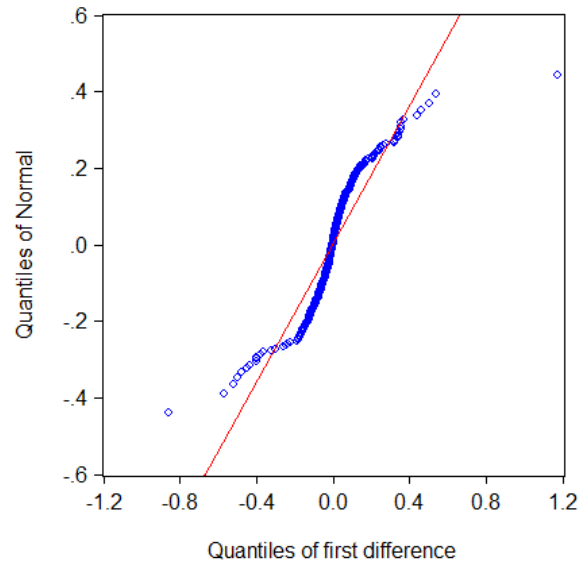


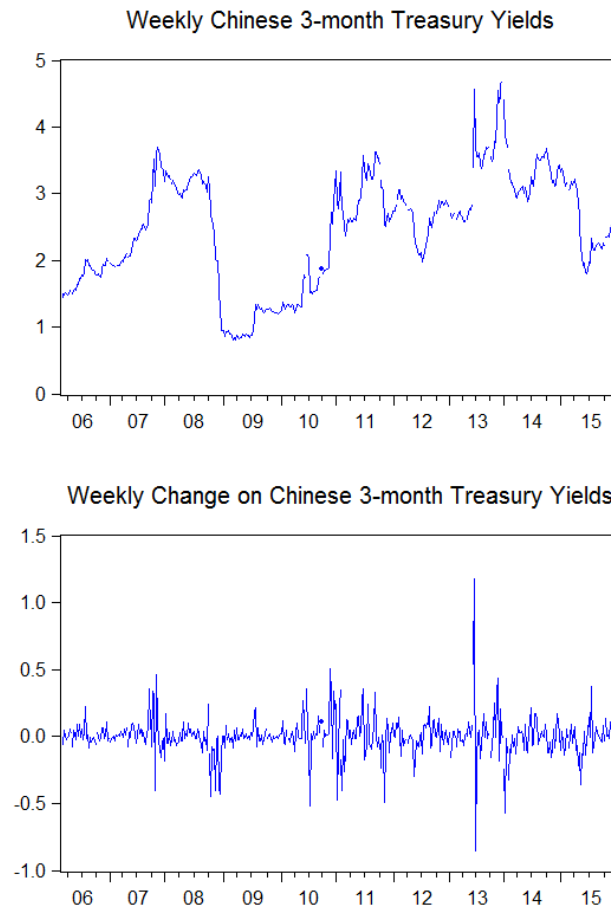
Figure V-2: QQ plot of change on Chinese 3-month treasury yields against Normal



In Figure V-3, the time evolution of three-month government bond yield and its first difference are plotted ranging from 2006 to 2015. Significant volatility clustering phenomenon is observed, since large moves follow large moves and small moves follow small moves on the change of interest rate. This result is consistent with the finding in data statistics. The level effect is also observable from the high volatility in 2008, 2011 and 2013 occurred accompanying the high levels of the short rate.

In addition, the interest rate behaves diversely in different time periods. We observe as least during three periods with high volatility which are 2007-2008, 2010-2011 and 2013. The observed high volatility of short-rate is considered to be driven by some events happening during the corresponding period. We try to account for them by relating them to economic or political events. The 2008 financial crisis had a great influence on China's economy and reduced the economic growth heavily. In 2011, the Shenzhen Composite Index fell by 32.86% and the Shanghai Composite Index fell by 21.68%. The Banking Liquidity Crisis occurred in 2013 when there was a severe liquidity squeeze across commercial banks. The inter-bank overnight repurchasing rate increased unprecedentedly to 30%, the highest level in the history. These events are widely considered to have impressive impacts on the volatility of short-term interest rate.

Figure V-3: 3-month Government Bond Yields and the Chang



According to the statistical summary of the data, four important features of the Chinese short rate can be concluded. 1) The Chinese short rate has a mean-reverting tendency. 2) Significant volatility clustering is observed and the distribution of rate change displays a heavy tail with positive excess kurtosis. 3) The short-term interest rate has a level effect. 4) At least two regimes exist in terms of the volatility level. These findings motivate this study to allow regime-switching in diffusion model of short-term interest rate.

V.2.2. Models of Short Term Interest Rate

In the literature, the short-term interest rate diffusion models are normally presented in the form of

$$dr_t = \mu(r_t; \theta)dt + \sigma(r_t; \theta)dW_t \quad (\text{VI.1})$$

where μ and σ^2 are drift and diffusion terms which determine the dynamics of short-rate. W_t is a standard Brownian motion and θ is a parameter to be estimated. Most existing studies assume the diffusion function to be $(\sigma r^\rho)^2$, where ρ is the elasticity of the volatility with respect to short-rate. It measures the sensitivity of interest rate volatility to the level of interest rate. In contrast, there is a debate on the assumption of drift component. In Vasicek (1977), CIR (1985) and CKLS (1992), the drift term is a linear function of interest rate level, while in CHLS (1997), Durham (2002) and Jones (2003) the drift term is specified as a nonlinear function in terms of interest rate level, to capture the weak mean-reversion feature when the short-rate is at middle level.

Since we are interested in examining the dynamics of Chinese short-rate, and both similarities and differences of it with the U.S., the CHLS (1997) model is employed in this study. The generalization of this framework allows us to compare a series models which are nested in one system. In CHLS (1997), the short-rate dynamics is governed by

$$dr_t = (\alpha_{-1}r_t^{-1} + \alpha_0 + \alpha_1r_t + \alpha_2r_t^2)dt + \beta r_t^\rho dW_t \quad (\text{VI.2})$$

The drift term $\alpha_{-1}r_t^{-1} + \alpha_0 + \alpha_1r_t + \alpha_2r_t^2$ is a nonlinear function of r_t . It captures stronger potency of the mean-reverting tendency at high and low levels of the interest rate and weaker mean reversion strength at the middle level. The first order derivative of the drift with respect to interest rate $(-\frac{\alpha_{-1}}{r^2} + \alpha_1 + 2\alpha_2r)$ is the mean-reversing speed which is a nonlinear function in terms of interest rate level. Thus, the mean reversion speed is allowed to vary at different short rate level. By contrast, the Vasicek, CIR and CKLS models define the drift component as $\alpha_0 + \alpha_1r_t$, where α_1 is the speed of return to its long-run mean $-\alpha_0/\alpha_1$. The mean-reversion intensity is assumed to be fixed at all the level of short rate.

The diffusion term governs the conditional volatility of the interest rate change. In CHLS, drift is defined in the same form as in CKLS model. As a function of r_t , it can capture the important feature that the interest rate is more volatile when the interest rate

level is high. This is the so-called level effect of short rate. The CIR model can also capture the level effect, since the drift term is still a function of r_t when $\rho = 0.5$. However, the Vasicek model assumes the conditional variance to be constant by imposing $\rho = 0$.

The single-factor diffusion models we employed in this study can be nested within the CHLS framework by imposing the restrictions $\alpha_{-1} = \alpha_2 = \rho = 0$ for Vasicek, $\alpha_{-1} = \alpha_2 = 0$ and $\rho = 0.5$ for CIR, and $\alpha_{-1} = \alpha_2 = 0$ for CKLS.

Based on the evidence from the U.S., the stochastic behaviour of short rate might change during different time periods due to change of monetary policy, financial crisis and other economic conditions. Thus, the existence of regime changes suggests regime-switching models instead of single-regime models. In order to capture the possible regime shifting in Chinese short rate, we extend the single-regime CHLS framework to regime-switching form by allowing both drift and diffusion terms to shift within two regimes.

In the Markov regime-switching CHLS model, the movement of short rate is governed by a stochastic differential equation in discrete version as

$$\Delta r_t = r_t - r_{t-1} = a_{-1,j}r_{t-1}^{-1} + a_{0,j} + a_{1,j}r_{t-1} + a_{2,j}r_{t-1}^2 + \beta_j r_{t-1}^{\rho_j} u_t \quad (\text{VI.3})$$

where r_t denotes the interest rate at time point t , and the parameters to be estimated are $a_{-1,j}$, $a_{0,j}$, $a_{1,j}$, $a_{2,j}$, β_j and ρ_j which are all regime-dependent according to j . In this study, we allow the short rate to shift within two regimes which are captured by the unobserved regime indicator j . It is assumed that the regime indicator follows a continuous time first order Markov chain with two states. u_t is the standard normal random variable.

The behaviour of changes in Chinese short rate observed in the last section motivates us to separate two regimes with low and high volatility respectively to characterize two different economic environments. The stochastic behaviour of short rate either with a low or high volatility can be captured at any time of point. However, the state of short

rate falls in can only be inferred but not be observed. The probability of staying in the low or high volatility regime can be estimated by using the above Markov regime-switching models. In the regime-switching framework, the conditional mean and conditional variance of the short rate enable the model to take different values according to two states of the regimes and the probabilities of the shifting between regimes are decided by the interest rate level. The conditional mean takes different speed of reverting to different long-run equilibrium means in each regime and the conditional variance in this model captures the level effect.

The same restrictions as in the single-regime framework are placed in equation (VI.3) to obtain the Markov regime-switching Vasicek, CIR and CKLS models. The models employed in this study are given in the following table.

Table V-2 Single-regime and regime-switching Models employed in this study

Single-regime short rate models	
Vasicek:	$r_t - r_{t-1} = a_0 + a_1 r_{t-1} + \beta u_t$
CIR:	$r_t - r_{t-1} = a_0 + a_1 r_{t-1} + \beta \sqrt{r_{t-1}} u_t$
CKLS:	$r_t - r_{t-1} = a_0 + a_1 r_{t-1} + \beta r_{t-1}^\rho u_t$
CHLS:	$r_t - r_{t-1} = a_{-1} r_{t-1}^{-1} + a_0 + a_1 r_{t-1} + a_2 r_{t-1}^2 + \beta r_{t-1}^\rho u_t$
Regime-switching short rate models	
Vasicek:	$r_t - r_{t-1} = a_{0,j} + a_{1,j} r_{t-1} + \beta_j u_t$
CIR:	$r_t - r_{t-1} = a_{0,j} + a_{1,j} r_{t-1} + \beta_j \sqrt{r_{t-1}} u_t$
CKLS:	$r_t - r_{t-1} = a_{0,j} + a_{1,j} r_{t-1} + \beta_j r_{t-1}^{\rho_j} u_t$
CHLS:	$r_t - r_{t-1} = a_{-1,j} r_{t-1}^{-1} + a_{0,j} + a_{1,j} r_{t-1} + a_{2,j} r_{t-1}^2 + \beta_j r_{t-1}^{\rho_j} u_t$

V.2.3. Maximum Likelihood Estimation of Regime-switching Models

As the latent regime indicator j is assumed to follow a first-order Markov process, the transition probability matrix can be obtained based on Hamilton (1988, 1989 and 1990).

$$\Pr[j = 1 | j = 1] = P \quad (\text{VI.4})$$

$$\Pr[j = 2|j = 2] = Q \quad (\text{VI.5})$$

$$\Pr[j = 2|j = 1] = 1 - P \quad (\text{VI.6})$$

$$\Pr[j = 1|j = 2] = 1 - Q \quad (\text{VI.7})$$

where $j = 1$ or 2 for regime 1 and regime 2 respectively. The Markov chain enables the current state to depend on the previous states, although it is unobservable. If the short rate is in state 1 at this moment, it can stay in state 1 or shift to state 2 in the next period. The probability of staying in state 1 is denoted by P as shown in equation (VI.4) and the probability of shifting to state 2 can be expressed as $1 - P$ in equation (VI.6). In contrast, if the current state is state 2, the dynamics of short rate may stay in state 2 or switch to state 1 in the next period. The probabilities are given in (VI.5) and (VI.7) as Q and $1 - Q$ for remaining and switching respectively.

We estimate the Markov regime-switching models by using maximum likelihood method as in Hamilton (1989, 1994) and Gray (1996). The log-likelihood function can be written as

$$\begin{aligned} L(\theta) &= \sum_{t=1}^n [\log\{f(\Delta r_t | \Phi_{t-1})\}] \\ &= \sum_{t=1}^n [\log\{\sum_{i=1}^2 f(\Delta r_t | j = i, \Phi_{t-1}) \Pr(j = i | \Phi_{t-1})\}] \end{aligned} \quad (\text{VI.8})$$

where the information set at time $t - 1$ is expressed as Φ_{t-1} . The conditional density of Δr_t generated from regime i at time t , follows a normal distribution denoted as

$$f(\Delta r_t | j = i, \Phi_{t-1}) \sim N(a_{-1,j}r_{t-1}^{-1} + a_{0,j} + a_{1,j}r_{t-1} + a_{2,j}r_{t-1}^2, \beta_j^2 r_{t-1}^{2\rho_j}) \quad (\text{VI.9})$$

with conditional mean $a_{-1,j}r_{t-1}^{-1} + a_{0,j} + a_{1,j}r_{t-1} + a_{2,j}r_{t-1}^2$ and conditional variance $\beta_j^2 r_{t-1}^{2\rho_j}$. $f(\Delta r_t | \Phi_{t-1})$ represents the density of Δr_t given the information set at $t - 1$ and is the average of $f(\Delta r_t | j = i, \Phi_{t-1})$ weighted at $\Pr(j = i | \Phi_{t-1})$ which is the so-called ex ante probability.

V.3. Empirical Analysis

In this section, both single-regime and regime-switching models nested in the CHLS framework are estimated using maximum likelihood method. The Ljung-Box statistic is employed for diagnostic testing. We compare nested models with same number of regimes by the likelihood ratio test. Due to the burden on computation, no formal test is applied on the second regime. Following most studies in literature, we try to find economic significance on the second regime and leave this for future study.

V.3.1. Parameter Estimations

In Table V-3, the parameter estimations of single-regime Vasicek, CIR, CKLS and CHLS models are displayed in columns 2-5. All the parameters are statistically significant in all single-regime models. The estimation of drift terms in Vasicek and CIR models indicates significant mean-reversing tendency of the short rate with negative \widehat{a}_1 . But the estimate of the reversion coefficient is positive in CKLS. Hong, Lin & Wang (2010) also find significant mean-reversion on 7-day repo rate in China. The elasticity of volatility ρ is about 0.82 in both CKLS and CHLS models compare to around 1.5 from studies based on the U.S.. The relatively lower elasticity of volatility indicates that the volatility of short rate is less sensitive to the interest rate level in China than that in the U.S.. This result is consistent with Hong, Lin & Wang (2010)'s finding. They obtain the estimate of ρ at 0.7595 in CKLS and 0.7720 in CHLS, which are very close to the value we find. In addition, the contribution of free the elasticity of volatility to be estimated explicitly rather than pre-set is limited, since the log-likelihood value only increases slightly from 300 to 311. Meanwhile the log-likelihood value stays the same by imposing nonlinear drift in CHLS over the assumption of linear drift in CKLS. Hong, Lin & Wang (2010) achieve similar indication on the limited contribution of incorporating nonlinear drift.

Table V-3 Parameter estimations in single-regime models

Single-regime

	Vasicek	CIR	CKLS	CHLS
a_{-1}	-	-	-	-0.0149*** (0.0000)
a_0	0.0341*** (0.000)	0.0388*** (0.0000)	0.2523*** (0.0000)	0.0424*** (0.0000)
a_1	-0.0104*** (0.000)	-0.0150*** (0.0000)	0.0946*** (0.0000)	-0.0175*** (0.0000)
a_2	-	-	-	0.0110*** (0.0000)
β	0.1422*** (0.0016)	0.0857*** (0.0011)	0.0609*** (0.0014)	0.0609*** (0.0009)
ρ	0	0.5	0.8174*** (0.0284)	0.8172*** (0.0237)
Log-likelihood	257.3865	300.2301	311.3718	311.3718

Notes: The asterisks *, ** and *** represent the 10%, 5% and 1% statistical significance levels respectively and the estimation standard errors are indicated in the parentheses.

The estimation results of two-regime Vasicek, CIR, CKLS and CHLS models are given in Table V-4. We find interesting features of Chinese short rate which cannot be captured in single-regime models.

First, two different regimes are identified with high volatility in regime 1 and low volatility in regime 2. The standard deviation in regime 1 is 0.27, 0.16 and 0.11 in three linear drift models, more than four times of those in regime 2. Both regimes show high persistence and regime 2 is much more persistent than regime 1. Although most of the drift coefficients are not statistically significant, they do provide interesting economic results. We find that regime 1 with higher volatility has a higher long-run mean while regime 2 with lower volatility has a relatively low long-run mean. In regime 1, the implied long-run means of the short rate are 3.04, 2.94 and 3.28 in Vasicek, CIR and CKLS models while those are 2.35, 2.53 and 2.67 in regime 2 respectively. The finding that all the estimates of reversion speed are negative is consistent with mean-reversion, but the evidence is weak due to the insignificance of the drift parameters. Hong, Lin & Wang (2010) find strong evidence on mean-reversion dynamics within each regime by

using 7-day repo rate in China. In contrast, studies based on the U.S. find that the short rate behaves as a random walk in the regime with low volatility and shows mean-reversing tendency in the regime with high volatility.

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Table V-4 Parameter estimations in regime-switching models

	Two-regime			
	Vasicek	CIR	CKLS	CHLS
$a_{-1,1}$	-	-	-	0.1382* (0.0727)
$a_{-1,2}$	-	-	-	-0.0112*** (0.0000)
$a_{0,1}$	0.1284	0.0781	0.0393	-0.2837**

	(0.0930)	(0.0667)	(0.0519)	(0.1433)
$a_{0,2}$	0.0249**	0.0152*	0.0112	-0.3019**
	(0.0097)	(0.0081)	(0.0083)	(0.1373)
a_{11}	-0.0423	-0.0266	-0.0120	0.2022**
	(0.0308)	(0.0261)	(0.0246)	(0.0807)
a_{12}	-0.0106***	-0.0060	-0.0042	0.2013***
	(0.0040)	(0.0038)	(0.0040)	(0.0768)
a_{21}	-	-	-	-0.0388***
				(0.0000)
a_{22}	-	-	-	0.0119***
				(0.0000)
β_1	0.2669***	0.1646***	0.1128***	0.1241***
	(0.0208)	(0.0131)	(0.0301)	(0.0293)
β_2	0.0597***	0.0404***	0.0306***	0.0325***
	(0.0030)	(0.0020)	(0.0077)	(0.0051)
ρ_1	0	0.5	0.9080***	0.7819***
			(0.2679)	(0.0.2418)
ρ_2	0	0.5	0.8156***	0.7199***
			(0.2439)	(0.1589)
P	0.8035***	0.7932***	0.7639***	0.7922***
	(0.0564)	(0.0638)	(0.0669)	(0.0647)
Q	0.9375***	0.9417***	0.9315***	0.9382***
	(0.0176)	(0.0169)	(0.0226)	(0.0205)
Log-likelihood	427.2148	443.3768	445.7107	451.9175

Notes: The asterisks *, ** and *** represent the 10%, 5% and 1% statistical significance levels respectively and the estimation standard errors are indicated in the parentheses.

Second, the estimates of ρ in regime-switching CKLS and CHLS are ranged from 0.7 to 0.9, which are very close to those in the single-regime ones. The introducing of regime shift does not have significant impact on the elasticity of volatility. However, most studies focus on the U.S. find that the sensitivity of short rate volatility to the interest rate level can be largely reduced by allow regime-shifting in diffusion models. The elasticity of volatility is found to be around 1.5 in single-regime models and 0.5 in

two-regime models based on the U.S. data.

In addition, the Ljung-Box tests to all the nested models are given in Table V-5 for serial correlation of the squared residuals over 1, 5, 10 and 15 lags. We find significant serial correlation of the squared residuals in all the single-regime diffusion models. Introducing nonlinear drift will enhance on describing the stochastic volatility, however it is not able to capture the volatility clustering in Chinese short rate. In contrast, the serial correlation of squared residuals is reduced dramatically in all the regime-switching models. The p-values shown in the last three columns of Table V-5, test for serial correlation in both regime-switching CKLS and CHLS models. This finding indicates that much of the stochastic volatility on Chinese short-term interest rate can be captured by both regime-switching CKLS and CHLS models.

Table V-5. Ljung-Box statistic for serial correlation of the squared residuals out to i lag

	Single-regime				Two-regime			
	Vasicek	CIR	CKLS	CHLS	Vasicek	CIR	CKLS	CHLS
LB_1^2	101.86 (0.000)	104.28 (0.000)	105.89 (0.000)	81.683 (0.000)	16.787 (0.000)	5.924 (0.015)	3.563 (0.059)	3.530 (0.060)
LB_5^2	105.88 (0.000)	108.45 (0.000)	109.85 (0.000)	85.193 (0.000)	20.085 (0.001)	10.630 (0.059)	10.913 (0.053)	8.136 (0.149))
LB_{10}^2	105.98 (0.000)	108.56 (0.000)	109.968 (0.000)	85.281 (0.000)	23.193 (0.010)	13.695 (0.187)	15.402 (0.118)	12.179 (0.273)
LB_{15}^2	108.51 (0.000)	111.08 (0.000)	112.449 (0.000)	87.912 (0.000)	24.658 (0.055)	15.761 (0.398)	17.511 (0.289)	14.107 (0.518)

Note: The p -values are given in parentheses

Although, the log-likelihood value has been largely increased when the conditional mean and variance is allowed to shift within two regimes, the log-likelihood values are quite close in the four regime-switching models.

V.3.2. Model Comparison

We have estimated eight short-term interest rate models including four single-regime and four regime-switching. As given in Table V-3 and Table V-4, the log-likelihood value increases when allow more flexibility in the parameters. But the model selection cannot be determined by purely comparing the log-likelihood value, since model contains more variables or parameters always has a larger log-likelihood value even if some variables in the model are lack of explanation power. Thus, we employ the likelihood ratio test which is the most widely used test among nested models.

The likelihood ratio test results are given in Table V-6. We do not reveal the likelihood ratio between single regime and regime-switching models. The statistical significance of the second state cannot be tested by employing the likelihood ratio test, because under the null of a single regime, the parameters responding to the second regime cannot be identified and the statistical distribution will no longer be chi-squared. Thus, we only compare models with same number of regimes. In addition, the Vasicek and CIR have the same number of parameters, thus the in-sample performance can be compared directly by log-likelihood value. The increase in log-likelihood value from Vasicek to CIR indicates better performance of CIR in favour of Vasicek, either in single-regime or two-regime framework.

According to the tests, all the single-regime models are rejected against the CKLS model. This indicates that removing restriction on the elasticity of volatility can improve the in-sample fitting. The hypothesis that the drift term is nonlinear (CHLS model) against the CKLS model where the drift is linear, cannot be rejected at 5% significance level. This result suggests that the linearity assumption on the drift component is of secondary importance and may lead to over parameterization. In contrast, the nonlinearity is of importance in the regime-switching framework. The CHLS outperforms all the other models when inserting the regime-shifting specification. It is consistent with the estimation results reported in Table V-4. Most drift coefficients in linear models are insignificant, whereas all the parameters show statistical significance in CHLS model.

Table V-6. Likelihood Ratio Tests

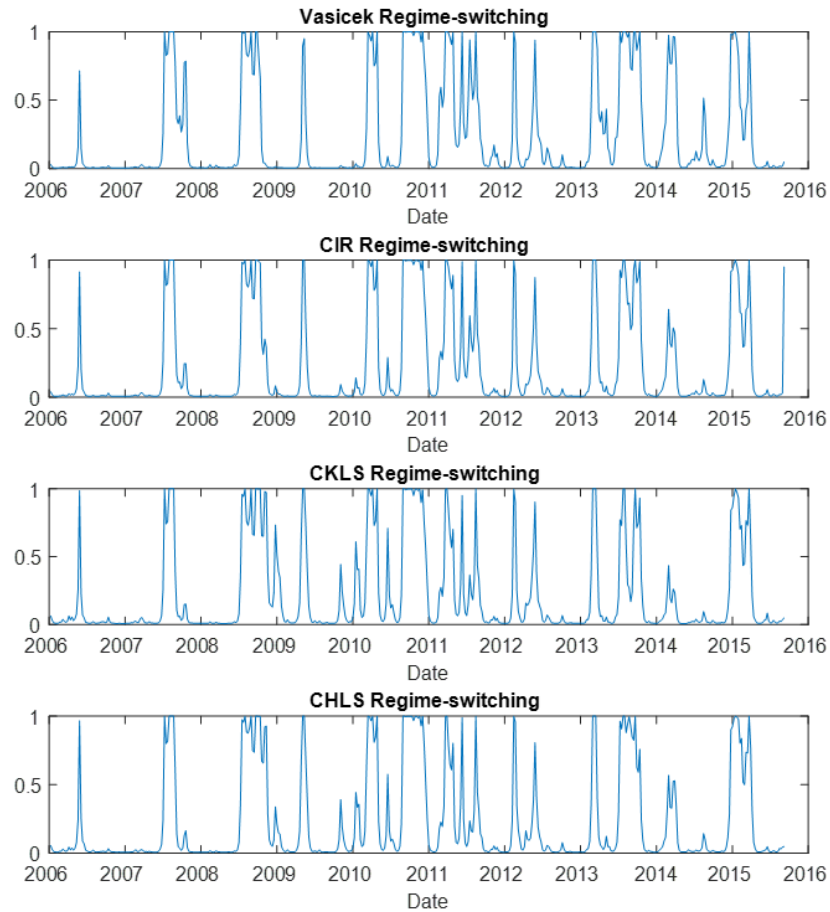
Single-regime		
	CKLS	CHLS
Vasicek	107.8106 (3.84)	107.8106 (7.81)
CIR	22.1234 (3.84)	22.1234 (7.81)
CKLS	-	0 (5.99)
Two-regime		
	CKLS	CHLS
Vasicek	36.9918 (5.99)	49.4054 (12.59)
CIR	4.6878 (5.99)	17.1014 (12.59)
CKLS	-	12.4136 (9.49)

Note: Chi-square distributions are given in parenthesis.

Models nested in this research have been compared with same regimes, but the existence of the second regime is not tested. As we have discussed, if we compare regime-switching CHLS with single-regime CHLS, there are six parameters unidentified under the null of single regime. In this case, the asymptotic theory cannot be employed and the LRT is no longer Chi-squared distributed. Thus, testing the number of regime is a challenging task. Hansen (1992) solved this problem by constructing a standardized LRT. But his method is considered to be extremely burdensome in computation even when on simple model. So following most empirical studies such as Hamilton and Susmel (1996) and Gray (1996), we do not apply any formal test on the second regime. Instead, we look at both LRT and the economic significance to obtain more confidence on the existence of the second regime. Ignoring the issue mentioned above, the LRT statistics strongly reject all the single regime models in favour of regime-switching models. To avoid the over-parameterization issue,

we appeal if the second regime is economically significant.

Figure V-4: Smoothed Probabilities in Regime 1 in Regime-switching Models



In Figure V-4, we plot the evolution of smoothed probability in regime 1 (with high volatility) for all the regime-switching models. The smoothed probability, defined as $\Pr[j = 1|\phi_T]$, shows if and when the regime shifts occurs. We cannot find the true regime the short rate in at each time point, only the probability of shifting is estimated. The overall patterns of the smoothed probability are the same in all the models, although slight differences are observed.

The periods fall to regime 1 with high variance can be explained with corresponding events. China's economy suffered from high inflation for a short period in 2006, due to the global oil spike. In August 2007, the CPI growth rate achieved its highest level in a decade as a result of dramatic growth on food price, especially pork. The next high

volatility period is driven by the global financial crisis occurred in 2008. The PBC raised RR eleven times in 2010 and 2011 to manage the excess liquidity and tame inflation. The high volatility regime in the second half of 2011 corresponds to the stock market crash in China. The following period is caused by the Banking Liquidity Crisis in 2013. The 2015 stock crash in China corresponds to the last regime 1 period. Therefore, the second regime is economically significant.

V.4. Conclusions

In this study, we examine the dynamic behaviour of Chinese short rate by comparing four one-factor diffusion models and four Markov regime-switching extensions. Based on the CHLS (1997) framework, Vasicek, CIR, CKLS and CHLS models are employed. Weekly inter-bank three-month treasury yields is used ranging from 2006 to 2015. All the models are estimated by maximum likelihood method following Hamilton (1988). We find that incorporating regime-switching can largely improve in-sample fitting to data and also help capture the volatility clustering and fatter tail. In addition, the regime-switching CHLS model provides the best in-sample performance among the others. The nonlinearity of drift term seems of importance on capturing the movement of Chinese short rate. Furthermore, the Chinese short rate is found more volatile than that in the U.S. and the volatility is less sensitive to the short rate level. In the regime-switching models, the Chinese short rate exhibits mean-reversion in each regime while the short rate in the U.S. is found mean-reversing in regime with high volatility and behaves as a random walk in regime with low volatility.

Our findings indicate that unlike market economies where the monetary policy rules are well understood and known, the volatility of the Chinese short rate first is somewhat higher in comparison and more importantly it cannot be explained to the same extent by the level as it happens in more mature and deep fixed income markets. This may reflect the need to explain the ‘rules’ conditioning policy, the evidence also suggests that the market view the rate as ‘stable’ and predictable to the extent that is strongly mean reverting in all regimes.

Furthermore, in this study no formal test is employed on the test of the second regime, since the existing method is burdensome for computation. Although the LRT and economic significance can provide a certain of confidence on the second regime, a more reasonable and easy to apply formal test will be a better choice. In addition, regard of using GARCH could be another possible suggestion for future work.

Chapter VI. Summary and Conclusions

This thesis studies the term structure of interest rates and the short-term rate volatility in China. It consists of an element of literature reviews on the term structure of interest rates and three empirical chapters. The empirical studies discuss the dynamics of yield curve, the interactions between yields and economy and the volatility in the short-term interest rate.

In the first empirical chapter, we employ the Fourier model to estimate the term structure of Chinese interest rates, following Moreno, Novales and Platania (2013). In order to establish whether the Chinese yield curve was affected by the 2008 global financial crisis, the in-sample fitting is conducted within two different sample periods, which are 2006-2015 and 2009-2015. Both the Vasicek and the Fourier extension models are found to provide significantly better fitting to the data by using the post-crisis sample than the whole sample. We contribute to the literature from three aspects. First, we find that the 2007 financial crisis has significant impact on the term structure of Chinese interest rates. Second, the Fourier model provides better approximation and prediction of the dynamics of Chinese yield curve than the Vasicek model, especially on the short end. Third, by comparing our study with Moreno, Novales and Platania (2013), we find that the financial crisis shock to the yield curve is more significant in the U.S. than in China. In addition, the Fourier assumption on long-run mean does help to capture the volatility of Chinese yield curves, as the Chinese yield curve is found to behave cyclically.

In the second empirical chapter, we construct and estimate the Nelson-Siegel form macro-finance model based on Chinese market, following Diebold, Rudebusch & Aruoba (2006). Interestingly, bidirectional causality is found, however the yield curve effect on the macroeconomy is relatively weak compared to the reverse influence. In the long-term horizon, both the inflation rate and real activity as approximate by industrial production, can explain more than 30 percent of the variation of yield curve. These results indicate flexibility and capacity of the dynamic Nelson-Siegel macro-finance model in describing the yield curve of an emerging market.

The last empirical chapter examines the dynamic behaviour of Chinese short rate in frame of CHLS (1997). Four one-factor diffusion models and four Markov regime-switching extensions are compared. We find that incorporating regime-switching can largely improve in-sample fitting to data and also help capture the volatility clustering and fatter tail. In addition, the least restricted regime-switching model performs best in-sample fitting among the others. The nonlinearity of drift term seems of importance on capturing the movement of Chinese short rate. Furthermore, the Chinese short rate is found more volatile than that in the U.S. and the volatility is less sensitive to the short rate level. In the regime-switching models, the Chinese short rate exhibits mean-reversion in each regime while the short rate in the U.S. is found mean-reversing in regime with high volatility and behaves as a random walk in regime with low volatility.

In conclusion, this thesis has captured many essential features of the interest rates dynamics in China, both over the entire yield curve and the short end. However, there are still some further work worthy to study in the future.

In the first empirical chapter, the Fourier model could be applied on derivative pricing and risk management of government bonds in the future. Additional flexibility could be added to the model by incorporating more terms of the Fourier series.

In the study of macro-finance Nelson-Siegel model, the no-arbitrage assumption was not imposed. Although the arbitrage-free restriction maybe undesirable when the yield curve is partially illiquid, to impose the condition of no-arbitrage opportunity to the Nelson-Siegel macro-finance model and detect if it can help on prediction is a suggestion for further research.

The last essay does not employ any formal test on the second regime, since the existing method is burdensome for computation. Although the LRT and economic significance can provide a certain of confidence on the second regime, a more reasonable and easy to apply formal test will be a better choice. This is a possible work for our future research.

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